THEORY OF THERMIONIC VACUUM TUBE CIRCUITS

THEORY OF THERMIONIC VACUUM TUBE CIRCUITS

BY

LEO JAMES PETERS

Assistant Professor of Electrical Engineering
The University of Wisconsin

FIRST EDITION
SECOND IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK 370 SEVENTH AVENUE
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1927

PREFACE

The purpose of this treatment of thermionic vacuum tube circuits is to develop conventions and methods which may be used in treating electrical networks and systems containing trielectrode devices. The topics and the circuits which might be discussed in a treatise on triode circuits are almost endless in number, and this book does not aim to cover them all. While most of the fundamental topics are covered, the main aim of the book is to impart to the reader a knowledge of fundamental theory and a familiarity with methods of attacking problems so that he can investigate systems and circuit arrangements other than those discussed in the book. It naturally follows then that the circuits and topics which are treated are those which best illustrate and fix in the mind of the reader the methods and conventions used in arriving at the performance of triode circuits.

The plan is to take the characteristic curves of the triode as a starting point and to develop the methods by which it is possible to predict from these curves the performance of the device in an electrical network. The book introduces four fundamental triode constants to treat those situations in which operation takes place over portions of the characteristic curves which are essentially straight lines. Two of these four constants were originally introduced into the discussion of triode circuit equations by Prof. Edward Bennett of the University of Wisconsin. One of these constants, the controlled grid conductance, is the ratio of the change in grid current to the change in plate voltage when the grid voltage is maintained constant. This ratio is relatively small and in many cases equal to zero for modern vacuum tubes which have a high vacuum. In an investigation, however, carried on in 1917 by Professor Bennett at the University of Wisconsin on the properties of open-air amplifiers, using the corona formation as a source of ions, this ratio was a relatively important one and it is therefore introduced for the sake of completeness of treatment.

The author follows the system of nomenclature for constants adopted by Professor Bennett. In this system of nomenclature

the ratio of the change in plate current to the resulting change in grid voltage when the plate voltage remains constant is called the controlled conductance of the plate by the grid, or briefly the controlled plate conductance. It is common practice elsewhere to call this constant the mutual conductance. This name is rejected because this quantity is in no accepted sense of the word a mutual one. If it were, it should equal the controlled grid conductance defined above. We have enough misnamed quantities in electrical engineering theory now without deliberately adding another to the list.

The third chapter introduces the idea of describing certain triode circuit phenomena as resistance neutralization. This idea is then used as a unifying thread to tie together the material presented in Chaps. II, IV, V, VI, and VIII. This method of presentation is a powerful aid in the establishing of a unity of viewpoint for the treatment of diverse phenomena.

I wish to express my indebtedness to Prof. Edward Bennett for the helpful suggestions which he has given during the writing of this book and also to Glenn Koehler for the experimental data which he furnished in connection with the calculations of the performance of amplifier circuits.

LEO JAMES PETERS.

MADISON, WISCONSIN,
July, 1927.

CONTENTS

CHAPTER IV

THE TRIODE AS A GENERATOR OF SUSTAINED OSCILLATIONS

CHAPTER V

BEHAVIOR OF RADIO RECEIVING SYSTEMS TO SIGNALS AND TO INTERFERENCE

CHAPTER VI

Triode Circuit Equations

CHAPTER VII

Modulation and Demodulation

CHAPTER VIII

Theory and Design of Amplifier Circuits

APPENDIX A

APPENDIX B

APPENDIX C

THEORY OF THERMIONIC VACUUM TUBE CIRCUITS

CHAPTER I

ELEMENTARY THERMIONIC THEORY

1. Introduction.

The **thermionic vacuum tube** is a device which may be used to control the motion of charges in an electric circuit or system. In mechanical systems a device used to control motions is called a valve, and the thermionic vacuum tube might appropriately be called the **thermionic valve.** This is the name generally used in England. The three-electrode thermionic vacuum tube or **triode** belongs to a class of valves in which the expenditure of a small amount of power on the control, or trigger, or power input element controls the flow of a large amount of power from a local source in the output element. The triode has the additional feature of effecting this control with inappreciable time lag. It is these two properties which give the triode a unique position among all other valves.

By suitable connections between the elements, the control of motions can be made such that the power available in the output element may be utilized in the following ways:

1. The power output may all be expended in the output element to accomplish some desired purpose; this is the case of **simple amplification.**

2. A part of the power output may be diverted back to the control element to supplement the power received from the external or original actuating agency in such a way as to increase the power-amplifying ratio; this is the case of **regenerative amplification.**

3. A portion of the power output may be diverted to the external system which supplies the power to the control element in such a manner as to cause the driving forces in the external system to deliver more power to the external system. The local source of power associated with the valve supplies a part only of the increased power which is expended in the external system. In the mathematical treatment of the system the valve constants enter the equations for the external circuit in the form of terms which **subtract** from the frictional or resistance terms. The valve when thus associated with the external agency is said to have the properties of a **negative resistance**; this is the case of **resistance neutralization.**

4. The power may be so diverted to the external system that any disturbance which is set up in the system may result in sustained oscillations (reciprocating motion) in a system in which the only driving force is the unidirectional force of the power source associated with the valve; this is the case of the generation of **sustained oscillations.**

Besides these four cases, the triode may be used to control the motion of charges in such a way as to accomplish three other important results:

1. The control of the motion of charges may be such as to produce low-frequency variations in the amplitude of a high-frequency current; this is the case of **modulation.**

2. The control of the motion of the charges may be such that a high-frequency current having an amplitude which varies at lower frequencies is modified in such a way that only the lower frequencies remain; this is the case of **demodulation.**

3. The control of the motion of charges may be such that an alternating motion of charges is converted into a pulsating or unidirectional one; this is the case of **rectification.**

The triode consists of a cathode, a plate or anode, and a metallic grid placed between the cathode and anode. These three elements may be mounted in the air, in a gas at atmospheric pressure, in a rarefied gas, or in a high vacuum. The cathode may be a heated metallic filament or other hot

body capable of emitting ions, a wire maintained at high potential in a gas so that corona forms around it, or any other source of ions. The anode generally consists of a metallic plate surrounding the cathode. It is charged in such a way as to attract to it the ions emitted by the cathode, thus causing the passage of an electric current between these elements. The grid, as the name signifies, consists of a wire mesh mounted between the cathode and the anode. It is charged in such a way as to control the motion of the ions from the cathode to the anode.

The discussion of triode theory naturally divides into two parts. The first part deals with the **properties of circuits** containing trielectrode devices, and the second part deals with the properties of the trielectrode device itself. In the first part, no inquiry is made into the nature of the processes by which conduction occurs in the space between the elements or into the *why* of the characteristic curves. The characteristic curves of the device are experimentally determined, and from these curves the conductances between plate and cathode or grid and cathode are computed. The properties of circuits into which such a device is connected are then analytically determined. This part of the discussion discloses the characteristics which are desirable and therefore should naturally come first. The second part deals with the *why* of the characteristic curves. It is an inquiry into the nature of the current flow from plate to cathode or grid to cathode, an inquiry into the manner in which the characteristics are affected by such variables as the nature and temperature of the filament, the gas pressure, and the applied potentials. This second part of the discussion discloses the principles by which trielectrode devices may be designed to have specified characteristics.

This book is primarily devoted to the first part of the discussion; that is, the book has for its object the development of methods and conventions which may be used to predict the behavior of the trielectrode device in an electric circuit or system when the characteristic curves are given. It is only incidentally concerned with those phenomena

inside the device which give rise to these characteristic curves.

1a. Some Types of Trielectrode Devices.

In the preceding section some of the forms which a trielectrode device may take were briefly pointed out. A more detailed description of a few of these devices will aid the reader in going over the theory presented in the following sections.

Figure 1 shows a schematic diagram of an experimental **corona amplifier.** W represents a wire about 6 feet long strung along the axis of a cylindrical wire grid G. The grid is about 1 inch in diameter. It is in turn surrounded by a metallic cylinder P, which will be called the plate. The plate is about 6 inches in diameter. These three elements are insulated from each other and immersed in a gas which may be air at atmospheric pressure. The grid is charged by the generator D_1 to a potential of about 8,000 volts above the wire W. The high electric intensity in the vicinity of the wire causes the air to ionize. This ionization of the air in the vicinity of the wire is called a corona discharge. A positive charge is placed on the plate P by means of the generator D_2. The magnitude of this charge is such as to raise the potential of the plate to about 30,000 volts above that of the wire. The charges on the plate and grid set up an electric field which causes the negative ions to move away from the corona wire. Most of the moving charges pass through the openings in the grid and go to the plate, thereby establishing a current between the plate and wire. This current may be read on the ammeter A_2. The charges attracted to the grid return to the corona wire through the ammeter A_1. The current which flows between the plate and wire may be controlled by changing the charge on the grid. The grid is the control element in the corona valve.

FIG 1.—Corona amplifier.

In order to give some idea of the magnitude of the currents which may be obtained in a corona amplifier, the following representative values are written down:

Potential of grid above wire..............	7,400 volts
Potential of plate above wire..........	28,000 volts
Plate-to-wire current.....................	2 milliamperes
Grid to wire current	1 milliampere

When the potential of the grid is increased by 500 volts, the plate-to-wire current is increased by 1.25 milliamperes and the grid-to-wire current is increased by 0.75 milliampere. The changes in power in the plate-to-wire circuit is (28,000)(0.00125) = 35 watts. The change in power in the grid-to-wire circuit necessary to cause this change in

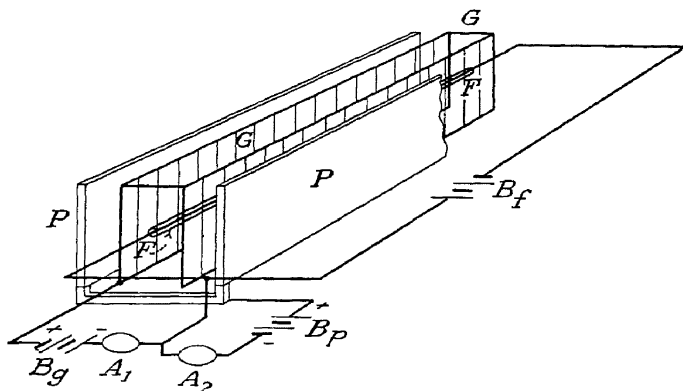



FIG 1*a*.—Glower oscillator

power in the plate-to-wire circuit is (7.9)(1.75) − (7.4)(1) = 6.43 watts. The power changes calculated above are given to illustrate the control action of the grid element.

Figure 1*a* is a schematic diagram of a **glower oscillator.** F represents a Nernst oxide glower about 2 inches long. A heating current may be passed through this glower by means of the battery B_f. G represents a wire grid placed about ⅛ inch on either side of the glower. The grid can be maintained at a given potential above the glower by charging it from the battery B_g. P represents two metallic plates spaced about ¼ inch from the grid. The plates can

be maintained at a given potential above the glower by means of the battery B_p. This device operates in air at atmospheric pressure.

The operation of the glower oscillator is as follows: The heated oxides which make up the glower element give off electrons. These electrons are attracted by the positive charges on the plate and grid, and a current is established between the plate and the glower and between the grid and the glower. Most of the charges pass through the openings in the grid and go to the plate, so that the current between the plate and the glower is larger than the current between the grid and the glower. The current between the plate and the glower can be controlled by changing the charge on the grid. The grid is the control element in the glower valve.

The magnitudes of the currents and voltages used in the operation of a glower oscillator, which has approximately the dimensions given above, have the following representative values:

Potential of grid above glower. . .	85 volts
Potential of plate above glower.	350 volts
Current between plate and glower	0.43 milliampere
Current between grid and glower	0.04 milliampere

Change in plate-to-glower current per volt change in plate potential $= 1.25 \times 10^{-6}$ amperes

Change in plate-to-glower current per volt change in grid potential $= 6.7 \times 10^{-6}$ amperes

Change in grid-to-glower current per volt change in grid potential 5×10^{-7} amperes

An examination of the above data brings out the following important facts relative to the control of the grid over the plate-to-glower circuit. A unit change in grid potential produces 5.4 times as much change in plate-to-glower current as a unit change in plate potential. The change in plate-to-glower current due to a change of 1 volt in grid potential is 13 times as great as the change in grid-to-glower current due to unit change in grid potential.

The corona amplifier and the glower oscillator were investigated at the University of Wisconsin by Prof. Edward Bennett in 1917. The corona amplifier could be made to function as an amplifier but would not oscillate. The possibilities of the device, however, when immersed in gases other than air, were not investigated. The open-air glower oscillator will function either as an amplifier or as an oscillator.

The three-electrode vacuum tube is by far the most important of the triode valves. It consists of a metallic filament similar to the filament in an ordinary incandescent lamp, a metallic plate which surrounds or partly surrounds the filament, and a wire grid interposed between the filament and plate. Figure 1*b* is a picture of a 5-watt-power tube with the glass container broken away. The three elements of the tube are visible in this picture. Figure 1*c* is the picture of a modern 250-watt-power tube.

FIG 1*b*—Five watt power tube with glass container removed.

Figure 1*d* is the picture of an early two-element tube used for a detector of radio signals. The tube pictured on the left in Fig. 1*e* is an early De Forest three-element tube, and the one pictured on the right is a tube which was in general use in radio receiving sets about 1920. These three pictures are given because of their historical interest.

In the three-element vacuum tube, the filament is heated to incandescence by passing an electric current through it. This heated filament emits electrons, and the motion of these electrons is controlled by placing charges on the plate and grid. A discussion of the emission of electrons by the filament and the control of the motion of these electrons is given in the following sections.

1b. The Emission of Electrons from Hot Bodies.

In all of the trielectrode valves described in the preceding section, the conduction of electricity in the valve itself takes

FIG. 1c.—250 watt power tube.

place through mediums which, under normal conditions, are very good insulators. In the last two devices the conduction takes place because one of the three electrodes supplies the medium charges which may be acted upon by the electric field set up by the charges on the other two electrodes. In the vacuum tube the element which supplies the charges to the evacuated space is the heated metallic filament. It is the purpose of this section to discuss in an elementary way the main features of the emission of electrons from heated metals.

FIG. 1*d*—Early two element tube

According to the electron theory of matter, each atom of an element consists of a positively charged attracting center, called the nucleus, about which revolve one or more negative charges called electrons. The aggregate charge of the electrons is equal to the net positive charge of the nucleus. By reason of the interactions between adjacent atomic systems, some electrons may temporarily escape from the control of the attracting nuclei. Such electrons behave like free negatively charged gas corpuscles in the interstices between the atoms until they again become attached to some atomic system. It is estimated that air contains from 1,000 to 5,000 free electrons per cubic centimeter, while the number of free electrons in copper is thought to be of the order of 10^{19} per cubic centimeter. Like gas molecules, these free electrons have a random velocity of thermal agitation which is a function of the temperature of the body.

The free electrons move readily in the interior of a substance but at the surface of a body they experience a very large force which tends to keep them within the body. In Fig. 2, *AA* represents the surface of a metallic conductor in a highly evacuated space. The curve *C* represents the

variation with distance of the surface force which tends to keep electrons within the body. This curve is not drawn to scale. Actually it falls practically to zero in a very short

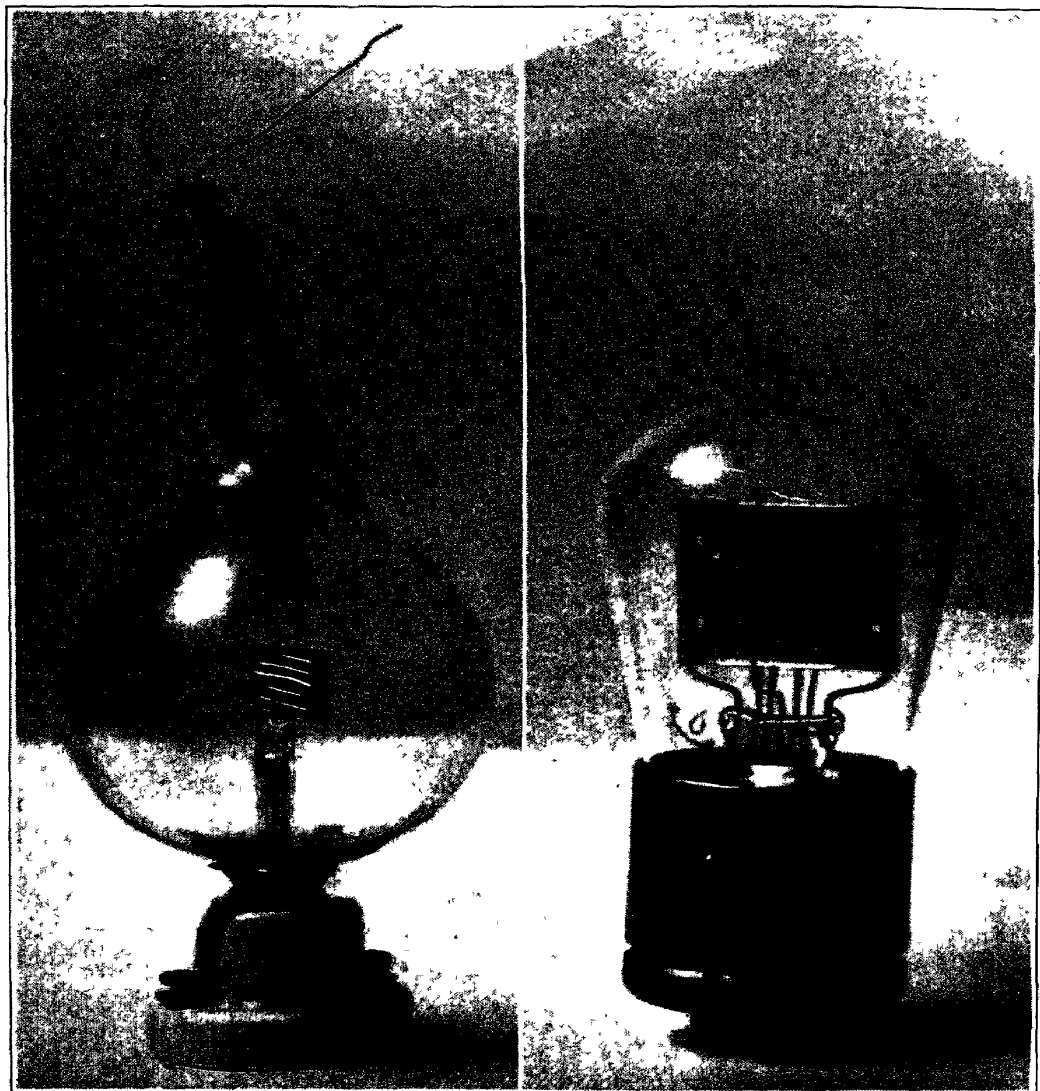



FIG. 1e—Left, early DeForest 3 element tube. Right, receiving tube, 1920

distance out from the body. The area under this curve out to any distance b represents the work which must be done on an electron to move it from the surface of the conductor

to b against the surface forces. Let this work measured in joules be represented by W_1. Let the mass of an electron in gram-sevens (10^7 grams) be represented by m and let its thermal velocity in centimeters per second at the surface AA be represented by v. Then if

$$\tfrac{1}{2}mv^2 = W_1,$$

FIG. 2.—Surface force on electrons.

the electron will be brought to rest by the surface forces at a distance b from the conductor and then will be attracted back into the conductor, provided W_1 is less than the total area under the curve C. Let the total area under the curve C represent an amount of work equal to W joules. Then if an electron is to overcome the surface forces completely, it must have a thermal velocity at the surface AA such that

$$\tfrac{1}{2}mv^2 > W.$$

The work which must be performed in order to move 1 coulomb of electrons out through the surface of a metal into evacuated space against the surface force is called the **electron affinity** of the metal. Since this work is measured in joules, the dimensions of the electron affinity are the same as those of electromotive force (joules per coulomb), and electron affinities are given in volts. The electron affinities of a few of the pure metals are given below.

Metal	Electron affinity in volts	Metal	Electron affinity in volts
Tungsten	4 52	Thorium	3 4
Platinum	4 4	Sodium	1 82
Silver	4 1	Potassium	1 53

The velocity which an electron must have in order to escape from a piece of tungsten into evacuated space can now be calculated. The charge on an electron is 1.591 $\times 10^{-19}$ coulombs. The mass of an electron is 9×10^{-28}

grams. The velocity must be such that the kinetic energy of the electron is equal to the electron affinity of the tungsten multiplied by the electronic charge in coulombs. The kinetic energy of an electron in joules is

$$\tfrac{1}{2}(9 \times 10^{-28})10^{-7}v^2 = 4.5 \times 10^{-35}v^2$$

In order to escape from the tungsten, the velocity of the electron in centimeters per second must be

$$v = \left[\frac{(4.52)(1.59 \times 10^{-19})}{4.5 \times 10^{-35}}\right]^{\frac{1}{2}} = 1.26 \times 10^{8} \text{ centimeters per second.}$$

The velocity which an electron must have in order to escape from the thorium into a vacuum is

$$v = \left[\sqrt{\frac{3.4}{4.52}}\right] 1.26 \times 10^{8} = 1.09 \times 10^{8} \text{ centimeters per second.}$$

It is thus evident that electrons escape more readily from thorium than from tungsten, and this accounts in part for the use of thoriated filaments in modern vacuum tubes.

At ordinary temperatures but relatively few electrons have the requisite velocity to escape through the surface of a metal. When the metal is heated to a high temperature, however, it would lose electrons at a rapid rate if reactions did not enter to put a stop to the loss.

After one electron has escaped through the surface, the metallic body is left with a net positive charge so that the next electron to escape through the surface must overcome in addition to the surface force the attraction of the positively charged body and the repulsion due to the electron which has already escaped. As electrons continue to escape from the body, an electrostatic field is built up which ultimately drives electrons back into the body at the same rate as they leave it. A heated metallic body is thus surrounded by an atmosphere of electrons, and a dynamic equilibrium exists between the body and the atmosphere of electrons such that electrons leave and reenter the body at the same rate. If the temperature of the body is raised, the equilibrium is

destroyed and the density of the atmosphere of electrons around the body increases until a new equilibrium condition is reached.

1c. Thermionic Currents through Evacuated Space.

In Fig. 2*a*, *A* represents a heated metallic plate in a vacuum. Let the plate *A* be maintained at zero potential by connecting it to ground. This heated plate is surrounded by an atmosphere of electrons so that close up to the plate an electron is repelled by this atmosphere towards the plate. In this same region a positive charge would be acted on by a force directed away from the plate. If distances are taken as positive when measured away from the plate, then the potential gradient close up to the plate must

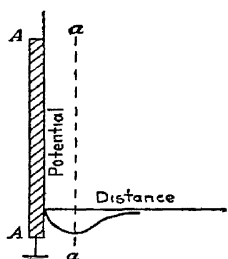



FIG. 2*a*.—Potential distribution near heated plate

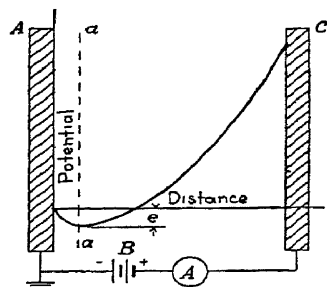



FIG. 2*b*.—Potential distribution between hot plate and charged cold plate

be negative. Further, the potential at the plate and at distances far removed from the plate must be zero. The curve which gives the potential as a function of the distance must have some such shape as that shown by Fig. 2*a*.

Let a cold plate be placed adjacent to the hot plate and let this cold plate be maintained at a potential of *E* volts above the hot plate by means of the battery *B*. The new arrangement is shown schematically in Fig. 2*b*. If the potential of the cold plate is not too high, the potential distribution in the vicinity of the hot plate has the same general shape as shown in Fig. 2*a*. In the case under consideration, however, the potential must have the value *E* at the cold plate. The potential distribution between the two plates must have the general shape shown by the curve in

Fig. 2*b*. The main feature of interest about this curve is the negative potential gradient to the left of the plane *aa* and the positive potential gradient to the right of the plane *aa*. In the region to the left of this plane the electrostatic force on an electron is directed towards the hot plate while to the right of this plane the force on an electron is towards the cold plate. That is, for an electron to reach the cold plate, it must be emitted from the hot plate with enough kinetic energy to carry it past the plane *aa* against the electrostatic forces which are directed towards the hot plate in the region to the left of this plane.

Let the negative potential of the plane *aa* be represented by e, and let the charge on an electron be represented by q. Then if an electron is to reach the cold plate, it must be

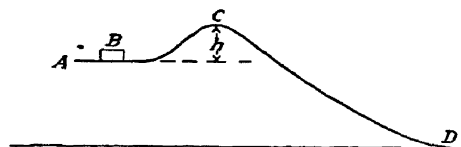



FIG 2*c*—Mechanical analogue of electrical conditions in Fig 2*b*

emitted from the hot plate with such a velocity that $\frac{1}{2} mv_n^2$ is at least equal to eq, in which v_n is the component of its velocity which is perpendicular to the plate AA. By velocity of emission is meant the velocity which the electron has after overcoming the surface forces of the hot metal plate.

The conditions necessary for an electron to pass from the hot plate to the cold plate in Fig. 2*b* are somewhat analogous to the conditions necessary to slide the block B of Fig. 2*c* over the hill ACD from A to C when friction is neglected. For the block to reach D it must be started from A with at least enough velocity to carry it to the top of the hill C. Upon passing the top of the hill, the force of gravity acts in such a way as to move the block from C to D. If the elevation of the point C is h centimeters above A, then the block must leave A with a kinetic energy at least equal to hmg in order to reach D.

Electrons are emitted from the hot plate with velocities ranging from zero up to high values, but only those electrons having a kinetic energy greater than eq can reach the cold plate. These negative charges moving from the hot plate to the cold plate constitute, under our conventions, a current from the cold plate to the hot plate through the evacuated space.

If the potential of the anode C is increased, the maximum negative potential e becomes smaller. There will then be a greater number of electrons emitted with the requisite kinetic energy eq to reach the cold plate, and the current will increase. Similarly, lowering the potential of the anode causes e to increase and the current to decrease. When the potential of the anode becomes so low that the kinetic energy required to reach the plane aa is greater than that possessed by any of the emitted electrons, the space current falls to zero. Any further reduction in the anode potential has no effect upon the space current. When the anode potential is increased to a value which makes the minimum potential e between the plates equal to zero, all of the emitted electrons reach the cold plate, and the strength of the space current cannot be made larger by increasing the anode voltage. It can, however, be increased by increasing the rate at which the hot plate emits electrons. The rate of emission of electrons by the hot plate can be increased by increasing its temperature. The current which flows when all of the emitted electrons reach the cold plate is called the **saturation current.** The saturation current is a function of the temperature of the hot plate.

By making use of assumptions and arguments drawn from the kinetic theory of gases, Richardson has deduced the following equation for the saturation current between the heated plate and the cold plate:

$$J \text{ (amperes per square centimeter of hot plate)} = a\sqrt{T}\ \epsilon^{-\frac{b}{T}}$$

in which a and b are constants, characteristic of each metal, and T is the absolute temperature of the hot plate.

The rate of emission of electrons from a hot metallic body is greatly affected by the presence of gas surrounding the

body and the condition of the emitting surface. Thus oxygen has the effect of greatly reducing the emission from platinum, and the presence of thorium as an impurity in tungsten greatly increases its electron emission at a given temperature.

1d. Energy Relations.

In Fig. 2*b* the cold plate *C* is maintained at a potential of *E* volts above the hot plate *A* by means of the battery *B*. The statement that *C* is maintained at a potential of *E* volts above *A* is equivalent to the statement that the forces of the electric field perform *E* joules of work on each coulomb of negative electricity transferred from *A* to *C*. If *q* represents the negative charge on an electron in coulombs, the work done by the forces of the electric field upon each electron as it passes from the hot plate to the cold plate is *Eq* joules. In a perfect vacuum, the motion of the electron is unrestrained and the work *Eq* must be transformed into an increase in kinetic energy of the electron. If the velocity of emission is neglected, then on reaching the cold plate the electron must have a velocity such as to satisfy the equation

$$\tfrac{1}{2}mv^2 = Eq$$

The velocity which an electron has acquired when it reaches the cold plate then is

$$v(\text{centimeters per second}) = \sqrt{\frac{2Eq}{m}}$$

Now the ratio of charge to mass for an electron is 1.768×10^{15} coulombs per gram-seven. Therefore the velocity which an electron has when it reaches the cold plate, neglecting the velocity of emission, is

$$v(\text{centimeters per second}) = 5.95 \times 10^7 E^{\frac{1}{2}}$$

If the potential of the cold plate above the hot plate is 1,000 volts, an electron reaches the cold plate with a velocity of 1.87×10^9 centimeters per second.

Upon reaching the cold plate, the electron shares its kinetic energy with the molecules and electrons of the cold

plate; that is, the work done on the electron by the electrical forces in the region between the plates is converted into heat energy at the cold plate.

In power tubes the electron bombardment is sufficiently great to heat the plate to incandescence. Tubes which handle large quantities of power are built so that the plate forms part of the container. This allows the plate to be cooled by immersing it in water.

2. Characteristic Curves of the Two-element Vacuum Tube. A circuit which may be used for studying the manner in which the current through a vacuum between a heated

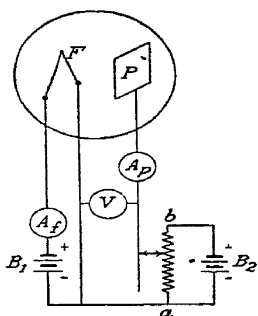



FIG. 3 — Connections for obtaining characteristics.

Plate Space Current
Plate Potential

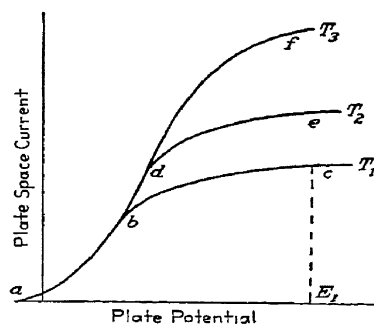



FIG 3*a* —Characteristic curves of two element tube.

metallic filament and a cold plate varies with changes in cold-plate potential and filament temperature is shown in Fig. 3. *F* represents a metallic filament which may be heated to a high temperature by passing a current through it by means of the battery B_1. This heating current will be called the **filament current.** Its magnitude may be read on the ammeter A_f. *P* represents the cold plate. It may be maintained at any desired potential (between limits) higher or lower than that of one end of the heated filament by means of the battery B_2 and the potentiometer *D*. The **plate potential,** that is, the potential of the plate relative to one end of the filament, may be read on the voltmeter *V*. The **plate current,** that is, the current which flows from the plate to the filament, may be read on the ammeter A_p.

If the filament temperature is maintained constant, at a value T_1, the plate current is given as a function of the plate

potential by a curve such as *abc* of Fig. 3*a*. This curve shows that the plate space current increases with increases in plate potential until the voltage E_1 is reached. Potentials in excess of this produce substantially no further increase in plate current. If the filament temperature is increased from T_1 to T_2, the plate current is given as a function of the plate voltage by the curve *ade*. For a filament temperature T_3, which is still greater than T_2, the current voltage curve is *adf*. These curves all have the same general shape. At the higher filament temperatures the satura-

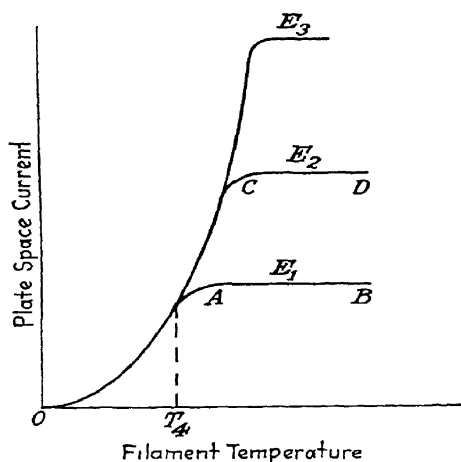



FIG 4.—Characteristic curves of two element tube

tion current has higher values and is reached at higher values of plate potential.

The curves given in Fig. 3*a* indicate that there is a range of plate potentials for which the plate space current is practically the same for the three temperatures T_1, T_2, and T_3. The curves of Fig. 4 give the plate current as a function of the filament temperature for three plate potentials. For temperatures less than T_4, the current is substantially the same for the three plate potentials considered. Over the portion *AB* of the curve *OAB*, the plate current is practically independent of the filament temperature. The curves for the other plate potentials show the same characteristic flattening out at the higher filament temperature. The general features of the conduction of currents in the

two-element vacuum tube as shown by the curves of Figs. 3*a* and 4 are in agreement with those given in the theoretical discussion of Secs. 1*d* and 1*e*.

3. Characteristic Curves of the Three-element Vacuum Tube.

It was pointed out in Sec. 1*c* that the presence of electrons in the region between the cold plate and the hot plate shown in Fig. 2*b* cause the potential to take on a maximum negative value in the vicinity of the hot plate. Due to the fact that electrons are emitted from the hot plate with a large range of velocities, the current between the two plates depends upon the maximum value of this negative potential.

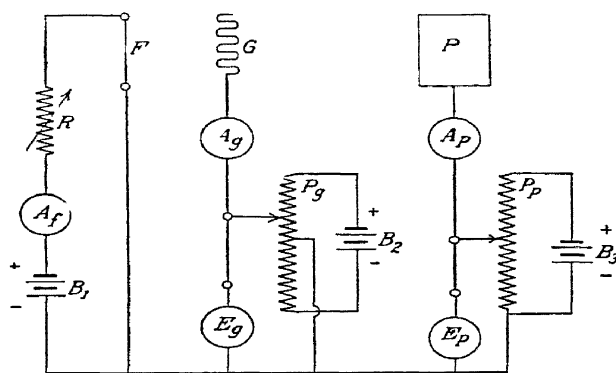



FIG 5.—Circuit for obtaining characteristics.

Now the value of this maximum negative potential may be controlled by placing a wire grid between the two plates and controlling the magnitude of a charge placed on this grid. Since the grid is closer to the hot plate than is the cold plate, a given potential impressed on the grid will have more effect upon the potential in the vicinity of the hot plate than will an equal potential impressed on the cold plate. Since the area of the wires comprising the grid can be made small compared to the area of the cold plate, the electrons can pass readily through its meshes to the cold plate. This grid may be used to control the current which flows between the two plates.

The circuit shown in Fig. 5 may be used to study the relation between the potentials applied to the grid and plate

and the resulting plate and grid currents. In the circuit of Fig. 5, F represents a metallic filament which can be heated by passing a current through it. The grid is represented by G. It can be maintained at potentials above or below the potential of one end of the filament by means of the battery B_2 and the potentiometer P_g. The current which flows from the grid to the filament and plate can be read on the

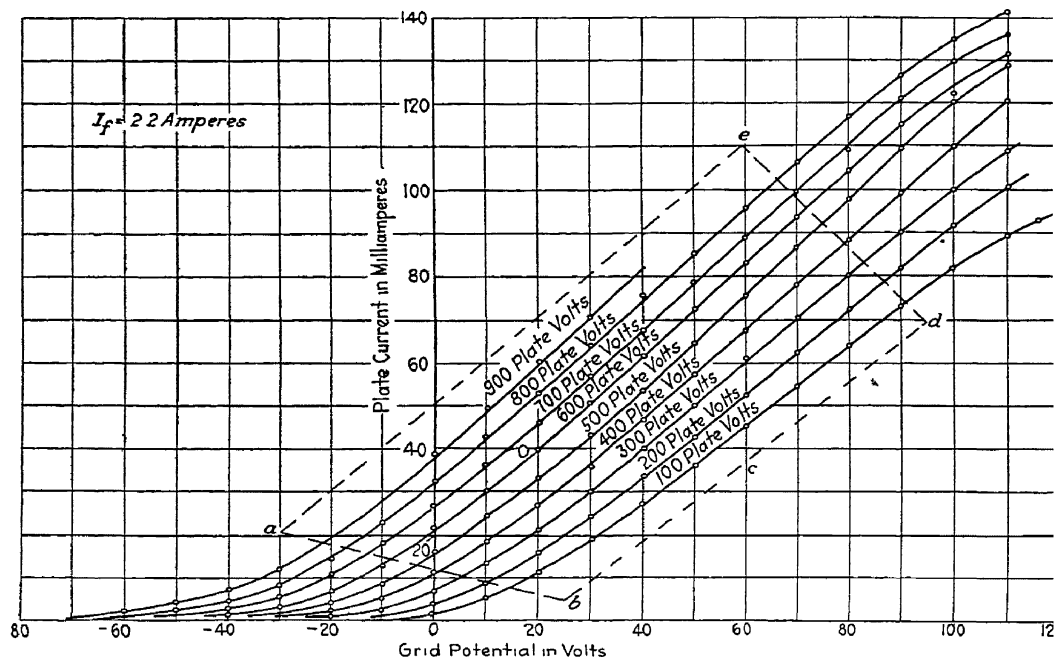



FIG. 6.—Characteristic curves for a 100 watt power tube.

ammeter A_g. This current will be called the **grid current.** The potential of the grid above that of one end of the filament can be read on the voltmeter E_g. This potential will be called the **grid potential.** P represents the cold plate. The **plate current,** that is, the current which flows from the plate to the filament and grid can be read on the ammeter A_p. The plate can be maintained at potentials above that of one end of the filament by means of the battery B_3 and the potentiometer P_p. The **plate potential** can be read on the voltmeter E_p.

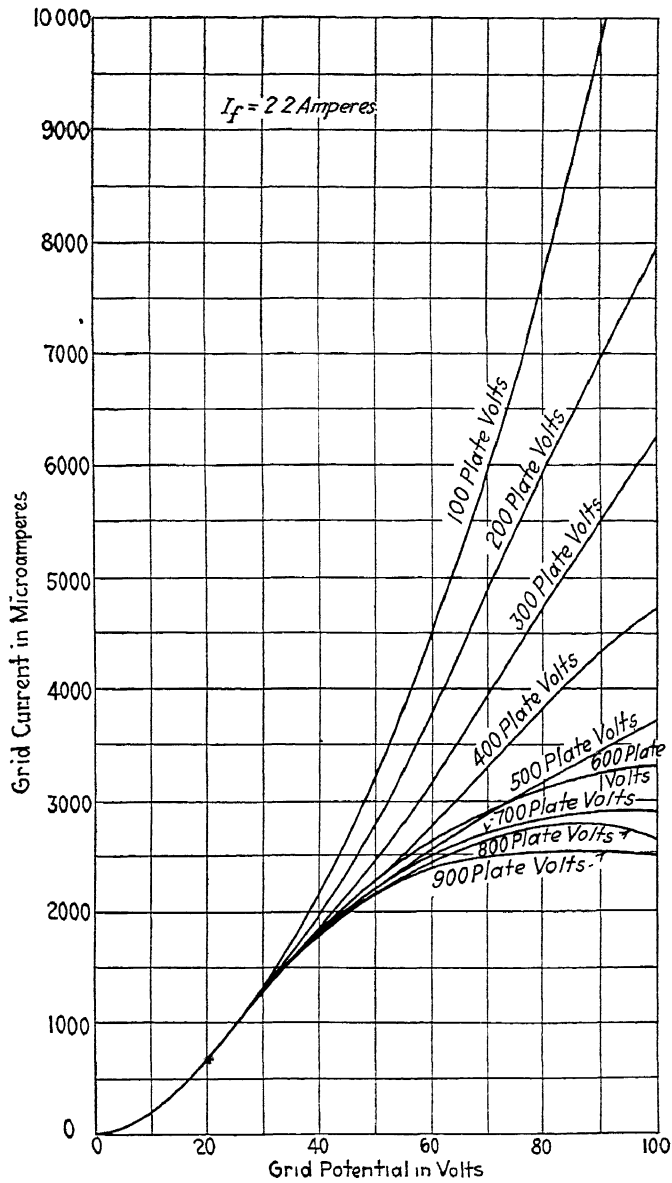



Fig. 7.—Characteristic curves of 100 watt power tube

With this circuit, data for the following sets of characteristic curves can be obtained. The first set consists of curves which give the **plate current** as a function of the **grid potential** for specified values of **plate potential.** Figure 6 gives such a set of curves for a 100-watt-power tube. The second set of curves gives the **grid current** as a function of the **grid potential** for specified values of **plate potential.** Such a set of curves for a 100-watt-power tube is given by Fig. 7. A third set of curves which sometimes is very useful is a set showing the **plate current** as a function of the **plate potential** for given values of the **grid potential.**

An examination of Figs. 6 and 7 brings out the following facts: The grid current is much smaller than the plate current; it may be reduced to zero by applying a negative potential to the grid. The change in plate current for a given change in grid potential is much larger than the change in plate current due to an equal change in plate potential. These properties give the triode its important amplifying characteristics.

In order graphically to represent the plate current as a function of the plate and grid potentials when the filament temperature remains constant, a three-dimensional domain must be used. In this three-dimensional domain the plot of the functional relation between the plate current and the plate and grid potentials would give a surface called the **characteristic surface** of the triode for the given filament temperature. The equation of this surface cannot be written down, but Van der Bijl has shown that it has the form

$$i_p = f(\mu v_g + v_p)$$

4. Conventions.

Before we proceed to a quantitative treatment of triode circuits, it is essential that a set of conventions be adopted so that an interpretation can be placed upon the analytical work. The conventions to be used in this treatment of triode circuits are given below.

The elements of the thermionic vacuum tube or triode now under consideration are a heated cathode F, an anode

or plate P, and a third electrode G, interposed between the cathode and the anode. In most of the circuits to be dealt with, these three elements are connected through auxiliary apparatus to a common point or bus. Therefore all circuits will be represented in a manner similar to the one shown by Fig. 8. All potentials of tube elements will be expressed relative to the common bus.

By the **grid current** we shall mean the current flowing through the evacuated space from the grid to the filament and plate. By the **plate current** we shall mean the current

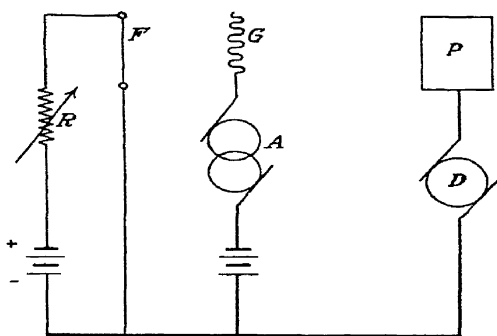



FIG 8

flowing through the evacuated space from the plate to the filament and grid.

In the consideration of circuits the following conventions will be adopted:

1. For convenience in specifying directions an arrow will be drawn in an arbitrarily selected direction along each branch of the network. The direction indicated by the arrow will be called the **arrow direction** in that branch.

2. Any symbol placed on the diagram to represent the current in a given branch will be understood to represent the algebraic value of the current in the **arrow** direction along the branch (This convention is nothing but a rapid and precise method of defining the symbol.)

3. Any symbol placed on the diagram to represent the electromotive force of specified forces in a given branch will be understood to represent the algebraic value of the electromotive force in the arrow direction along that branch.

(The known electromotive forces of batteries or generators are usually indicated by writing the numerical value on the diagram with separate polarity marks to indicate directions.)

4. Any symbol, as q, placed on the diagram to represent the charge in a condenser will be understood to represent the algebraic value of the charge on that electrode which receives positive charge when the current in the arrow direction along the branch is positive. In other words the symbol q is so defined that the following relation exists between q and i, when i represents the current in the arrow direction along the branch:

$$i = +\frac{dq}{dt} \qquad \text{or} \qquad q = +\int idt$$

5. The magnetic flux-linkage of a given circuit will be represented by the symbol $(+\Lambda)$. A positive numerical value will be assigned to (Λ) if the linkage of the circuit is the same in direction as that which would be caused by the flow of current in the arrow direction in that circuit.

6. The mutual inductance between two circuits will be represented by the symbol $(+M)$. A positive numerical value will be assigned to M if a current in the arrow direction in circuit 1 gives rise to a flux-linkage in circuit 2 which is in the same direction as the flux-linkage which would be caused by a current in the arrow direction in circuit 2.

7. The mutual elastance between the condensers in two circuits will be represented by the symbol $(+S_m)$. A positive numerical value will be assigned to S_m if the segregation of charge resulting from the flow of current in the arrow direction in circuit 1 gives rise to a potential difference between the plates of the condenser in circuit 2 of the same sign as that which would be caused by the segregation resulting from the flow of a current in the arrow direction in circuit 2.

8. The arrow direction in the grid and plate spaces will always be taken so that the arrow points from the grid to the filament and from the plate to the filament, respectively.

9. The e.m.f. of resistance in the arrow direction is $-Ri$, or in complex notation $-R\mathbf{I}$.

10. The e.m.f. of self-inductance in the arrow direction is $-L\frac{di}{dt}$, or in complex notation $-j\omega L\mathbf{I}$.

11. The e.m.f. of mutual inductance in the arrow direction is $-M\frac{di}{dt}$, or in the notation of the complex algebra $-j\omega M\mathbf{I}$. The sign of M is to be determined from convention 6.

12. The e.m.f. of capacitance in the arrow direction is $-\frac{q}{C} = -\frac{1}{C}\int idt$, or in complex notation $-\frac{\mathbf{I}}{j\omega C} = \frac{j\mathbf{I}}{\omega C}$.

13. Complex quantities will be designated by boldface Roman capital letters. The absolute value of these complex quantities will be designated by the same letter in italic capitals.

14. The instantaneous value of quantities which vary with the time will be represented by symbols printed in lower-case italic type.

4a. Definition of Triode Constants.

Consider the circuit of Fig. 8. Let the tube in this circuit have the characteristic curves given in Figs. 6 and 7. Let the potentials of the battery in the grid circuit and the generator in the plate circuit be adjusted so that operating conditions are represented on the curves of Fig. 6 by the point *o*. That is, the generator *D* maintains the plate at a potential of 600 volts above the negative end of the filament, and the battery in the grid circuit maintains the grid at a potential of 20 volts above the negative end of the filament. There is a continuous current of 40 milliamperes flowing in the plate circuit. This current is represented in Fig. 9 by the horizontal straight line marked 40×10^{-3} amperes. The plate space current as a function of the time is given by the equation

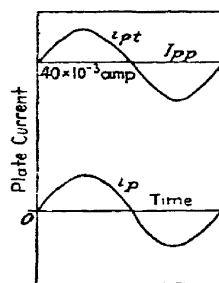



Fig. 9

$$I_{pp} \text{ (amperes)} = 40 \times 10^{-3} \qquad (1)$$

The point o will be called the **operating point on the characteristic surface of the tube,** or for short, the **operating point.**

Now let the alternator A in the grid circuit be started. Let this alternator deliver a terminal voltage given by the equation

$$e_g = 10 \sin \omega t \tag{2}$$

The total grid potential is then given by the equation

$$e_{gt} = 20 + 10 \sin \omega t \tag{3}$$

That is, the grid potential varies from 10 to 30 volts, while the plate potential remains constant at a value of 600 volts.

From the curves of Fig. 6 it is found that the plate current has a value of 30 milliamperes when the grid potential equals 10 volts. It has a value of 50 milliamperes when the grid potential is 30 volts and the plate potential is 600 volts. That is, the plate current varies from 30 to 50 milliamperes.

Over the range of variable grid potentials under consideration a straight-line relation exists between the grid potential and the plate current. If the capacitances between the tube electrodes are neglected (the effects of these capacitances are treated in detail in Chap. VIII), experience has shown that changes in plate current follow changes in grid potential without appreciable time lag. It follows, therefore, that the plate current is given by the equation

$$i_{pt} \text{ (amperes)} = (40 + 10 \sin \omega t)10^{-3} \tag{4}$$

This current is represented in Fig. 9 by the curve marked i_{pt}.

The variable part of the plate current is

$$i_p \text{ (amperes)} = 10 \times 10^{-3} \sin \omega t \tag{5}$$

This current is represented in Fig. 9 by the curve marked i_p.

The ratio of the variable plate current to the variable grid potential is

$$G_{cp} = \frac{i_p}{e_g} = \frac{10 \times 10^{-3}}{10} = 10^{-3} \tag{6}$$

That is, the ratio of the variable plate current to the variable grid voltage is a constant. It is equal to the slope of the 600-plate volt curve of Fig. 9 at the operating point *o*.

Let the triode under consideration be any triode. Let the operating point be so chosen that the characteristic surface in its vicinity is substantially a plane. Let a variable potential e_g be applied to the grid. Let the plate potential be constant. The potential e_g may be any function of the time, but its maximum value must be such that the plate space current varies only over the portion of the characteristic surface which is substantially a plane. The ratio of the variable plate current to the variable grid potential is a constant independent of the time. That is,

$$\frac{i_p}{e_g} = G_{cp} \tag{7}$$

The constant G_{cp} is the slope of the characteristic surface at the operating point measured in a plane for which the plate potential is constant. On a set of characteristic curves such as the ones given in Fig. 6, G_{cp} is the slope of the curve of constant plate potential which passes through the operating point. The slope is to be measured at the operating point.

Since the constant represented by G_{cp} is the ratio of a current to a potential, it has the dimensions of a conductance. It will be called the **controlled conductance of the plate by the grid,** or, for short, the **controlled plate conductance.**

That is to say, the **controlled plate conductance** (symbol G_{cp}) is defined to be equal to the ratio of the variable component of the plate current to the variable component of the grid potential when the plate potential remains constant. The defining equation of the controlled plate conductance is

$$G_{cp}\ (\text{mhos}) = \frac{i_p\ (\text{amperes})}{e_g\ \ (\text{volts})};\ (e_p = 0) \tag{8}$$

From considerations analogous to those used in defining the controlled plate conductance, let the following definitions be made:

The **plate conductance** (symbol G_p) is defined to be equal to the ratio of variable component of the plate current to the variable component of the plate potential when the grid potential remains constant. The defining equation of the plate conductance is

$$G_p \text{ (mhos)} = \frac{i_p \text{ (amperes)}}{e_p \text{ (volts)}}; \ (e_g = 0) \tag{9}$$

The **controlled conductance of the grid by the plate,** or, for short, the **controlled grid conductance** (symbol G_{cg}) is defined to be equal to the ratio of the variable component of the grid current to the variable component of the plate potential when the grid potential remains constant. The defining equation of the controlled grid conductance is

$$G_{cg} \text{ (mhos)} = \frac{i_g \text{ (amperes)}}{e_p \text{ (volts)}}; \ (e_g = 0) \tag{10}$$

The **grid conductance** (symbol G_g) is defined to be equal to the ratio of the variable component of the grid current to the variable component of the grid potential when the plate potential remains constant. The defining equation of the grid conductance is

$$G_g \text{ (mhos)} = \frac{i_g \text{ (amperes)}}{e_g \text{ (volts)}}; \ (e_p = 0) \tag{11}$$

In the defining Eqs. (8), (9), (10), and (11) i_p, e_p, i_g, and e_g represent the **variable components** of the plate current, the plate potential, the grid current, and the grid potential, respectively. Thus the condition that the plate potential remain constant at the value which it has at the operating point is expressed by the relation $e_p = 0$.

It should be realized that G_g and G_{cg} are not truly constants because, in general, a linear relation does not exist between the grid current, the plate potential, and the grid potential. In many cases operating points for triodes are chosen so that both of these constants are equal to zero.

Table I gives the constants of a number of vacuum tubes and also the constants of a typical corona amplifier and an open-air glower oscillator.

TABLE I —TRIODE CONSTANTS

Type	Conductances in micro-mhos				Continuous voltages		Peak alternating voltages		μ
	G_{cp}	G_p	G_{cg}	G_g	E_{pp}	E_{gp}	E_p	E_g	$\frac{G_{cp}}{G_p}$
5-watt VT 2.	1,700	250	0	16	200	0	50	5	6 8
100-watt high-vacuum tube	1,225	80	−0 39	34 5	500	+40	100	10	15 3
250-watt high-vacuum tube	3,650	170	+1 05	240	+600	+20	100	10	21 5
UV-200	1,025	350	0	0	20	−4	10	2	2 93
UV-201	335	56	0	2 6	20	0	10	2	6
UV-201	442	60	0	0	40	0	10	2	7 3
UV-201-A	740	92 5	0	0	90	0	10	1 25	8
C-12	300	41	0	0	60	0	10	1 25	7 3
C-299	200	32	0	0	60	0	10	1 00	6 25
VT 5	240	40	0	4	50	0	10	1 25	6 0
C-302	1,750	200	+1	75	400	0	50	10	8 75
Corona amplifier	2 48	0 076	−0 032	0 152	$2\ 8 \times 10^4$	$7\ 4 \times 10^3$	.	..	32 6
Corona amplifier	4	0 6	−0 013	0 617	$2\ 1 \times 10^4$	7×10^3		....	6 66
Open-air glower	6 7	1 25	−0 045	0 5	350	85	.	...	5 35

Problems

1. From the characteristic curves given by Figs. 6 and 7 determine the constants G_p, G_{cp}, G_g, and G_{cg} for the following conditions:

D-c grid voltage	20
D-c plate voltage	500
A-c grid voltage	10 (peak value)
A-c plate voltage	100 (peak value)

Determine the ratios $\frac{G_{cp}}{G_p}$

What conclusions can you draw from this ratio?

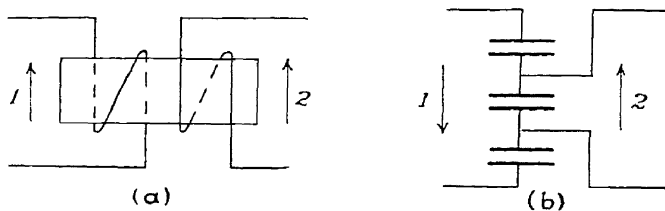



FIG 23

2. In Fig. 23*a*, is M positive or negative and in Fig. 23*b*, is the capacity coupling positive or negative? Give complete reasons for your answers based on the conventions of Chap. I.

CHAPTER II

ELEMENTARY AMPLIFIER THEORY

5. The Use of Triode Constants.

In the preceding chapter, four fundamental triode constants were introduced and defined. The present chapter is essentially an introduction to the methods of applying these constants to arrive at the properties of triode circuits.

Consider the triode circuit shown in Fig. 10. A_1 and A_2 are two potentiometers. Let the characteristic curves for the tube in the circuit be the straight lines given in Fig. 11. Let the battery B_1 have a terminal e.m.f. of 5 volts and the battery B_2 have a terminal e.m.f. of 200 volts. The point a in Fig. 11 is located on the characteristic curves so that the grid potential is 5 volts and the plate potential is 200 volts. This point is the operating point. When the potential across the potentiometers is zero, the plate current is given by the ordinate ab = 15 milliamperes. Let the potentiometer A_2 be kept at the zero setting, and let the potential across A_1 be increased to 10 volts. The new conditions in the circuit are that the grid potential is $5 + 10 = 15$ volts, the plate potential is $200 + 0 = 200$ volts, and conditions in the circuit are represented in Fig. 11 by the point c. The plate current has increased from 15 to 25 milliamperes. The increment in plate current is cd = 10 milliamperes. The following relation may be written down:

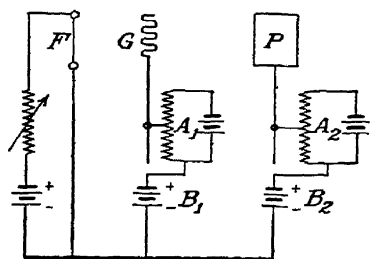



FIG 10.

$$cd = \frac{cd}{ad}ad \qquad (1)$$

Now $\frac{cd}{ad}$ is the slope of the 200-plate volt curve through the operating point and is therefore equal to the controlled plate conductance G_{cp}. The term *cd* is the increment in plate current and *ad* is the increment in grid potential. If

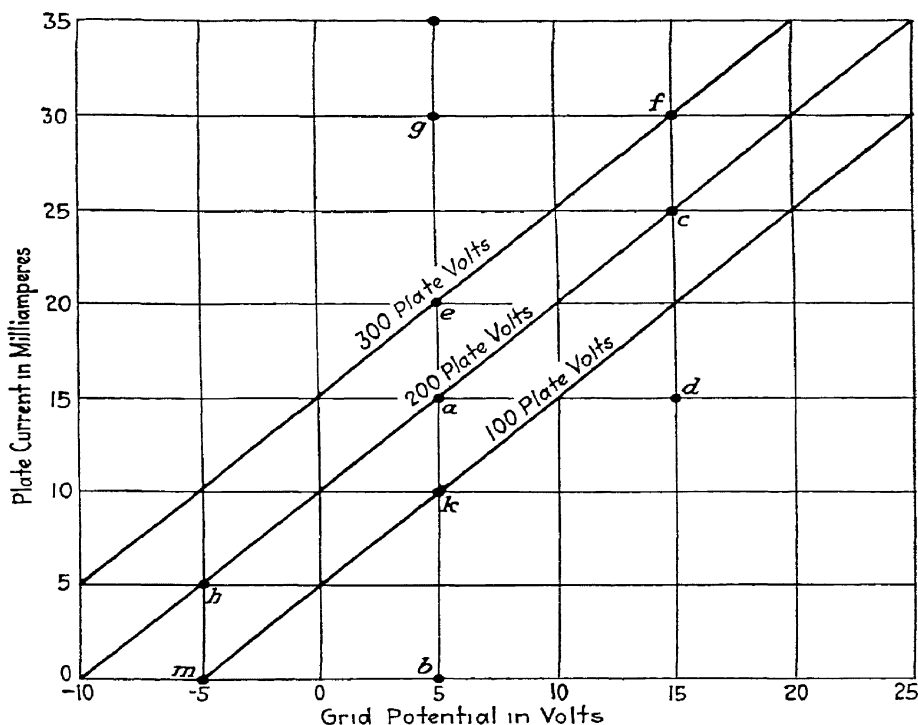



FIG. 11

Δ is used to denote an increment in the quantity following it, Eq. (1) may be written as

$$\Delta I_{p1} = G_{cp}\Delta E_g \tag{2}$$

Now let the potential of the potentiometer in the plate circuit be increased from 0 to 100 volts. The grid potential is 15 volts and the plate potential is 300 volts. The plate current has gone up from *c* to *f*, or from 25 to 30 milliamperes. For this case we may write the relations

$$\Delta I_{p2} = \frac{\Delta I_{p2}}{\Delta E_p}\Delta E_p \tag{3}$$

The ratio of ΔI_{p2} to ΔE_p is the slope of the characteristic surface measured in a plane for which the grid voltage is constant. It is equal to the plate conductance of the triode. Equation (3) may then be written as follows:

$$\Delta I_{p2} = G_p \Delta E_p \tag{3a}$$

The total increment in plate current due to both potentiometers is

$$\Delta I_p = ag = dc + cf$$

$$\Delta I_p = \Delta I_{p1} + \Delta I_{p2} = G_{cp}\Delta E_g + G_p \Delta E_p \tag{4}$$

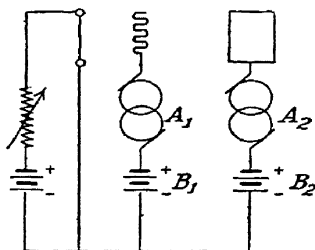



FIG. 12

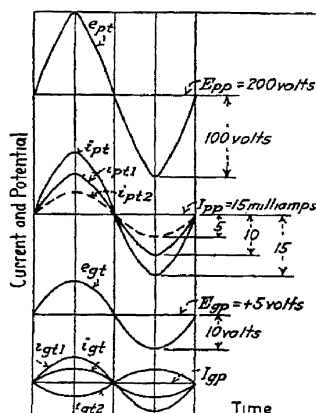



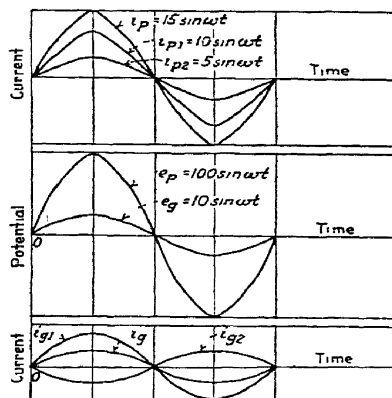
FIG. 13

In Eq. (4) G_{cp} is calculated at the point a of Fig. 11, and G_p is calculated at the point c. If the characteristic curves are straight lines, however, these two conductances are independent of the points where the calculations are made and G_{cp} and G_p may be calculated at the operating point a.

Now replace the potentiometers shown in Fig. 10 by alternators so that the circuit under consideration is the one shown in Fig. 12. Let the characteristic curves of the tube in the circuit of Fig. 12 be the straight lines shown in Fig. 11. B_1 delivers a terminal e.m.f. of 5 volts and B_2 delivers a terminal e.m.f. of 200 volts so that a is again the operating point. Let the terminal e.m.f. of the alternator in the grid circuit be equal to 10 sin ωt volts, and let the terminal e.m.f. of the alternator in the plate circuit be

zero. Under the conditions stated above, the plate potential has a constant value of 200 volts while the grid potential varies periodically from -5 to $+15$ volts. That is, operation takes place along the 200-plate volt curve from h to c. The plate current is composed of two parts, a continuous current of 15 milliamperes and a sine current having a peak value of $ac = 10$ milliamperes. In Fig. 13, the steady grid potential is represented as a function of the time by the line marked E_{gp}. The total grid potential is represented by the curve marked e_{gt}. The steady component of the plate current is represented by the line marked I_{pp}. The total plate current is represented by the curve marked i_{pt1}. In Fig. 13a, the variable component of the grid potential is represented by the sine wave labeled e_g. The variable component of the plate current is represented by the curve marked i_{p1}. The ratio of i_{p1} to e_g is, by definition, equal to the controlled plate conductance; so we may write

FIG 13a.

$$i_{p1} = G_{cp}e_g = G_{cp}10 \sin \omega t \qquad (5)$$

Now let the alternator in the grid circuit be stopped and let the alternator in the plate circuit deliver a terminal e.m.f. equal to 100 sin ωt volts. The grid potential remains constant at $+5$ volts while the plate potential varies periodically from 100 to 300 volts. In Fig. 11, operation takes place from k to e, and the plate current varies from 10 to 20 milliamperes. In Fig. 13 the continuous plate potential is represented by the line marked E_{pp}. The total plate potential is represented by the curve labeled e_{pt}. The total plate current is represented by the curve marked i_{pt2}. In Fig. 13a, the variable component of the plate potential is portrayed by the sine wave marked e_p and the variable component of the plate current is given by the sine curve labeled i_{p2}. The ratio of i_{p2} to e_p is, by definition,

equal to the plate conductance. We may, therefore, write the equation

$$i_{p2} = G_p e_p = G_p 100 \sin \omega t \tag{6}$$

Now let the alternator in the grid circuit deliver a terminal e.m.f. equal to 10 sin ωt, and let the alternator in the plate circuit deliver a terminal e.m.f. equal to 100 sin ωt. When the grid potential has a value of -5 volts, the plate potential is equal to 100 volts, and when the grid potential has a value of $+15$ volts, the plate potential is equal to 300 volts. On the curves of Fig. 11, operation takes place from m to f and the plate current varies periodically from 0 to 30 milliamperes. The total plate current is represented in Fig. 13 by the curve marked i_{pt}. In Fig. 13a the variable component of the plate current is represented by the sine wave marked i_p. Because the characteristic curves of the triode are straight lines, the curve i_p is the sum of the sine curves i_{p1} and i_{p2}. We, therefore, may write the equation

$$i_p = i_{p1} + i_{p2} = e_g G_{cp} + e_p G_p \tag{7}$$

In Fig. 13, I_{gp} represents the steady grid current at the operating point, and i_{gt1} represents the total grid current when the grid alternator is running and the plate alternator is stopped. The curve marked i_{gt2} represents the total grid current when the grid alternator is stopped and the plate alternator is running. The current curve i_{gt2} is shown 180 degrees out of phase with i_{gt1} because, in general, the grid current decreases when the plate potential increases; that is, G_{cg} is generally a negative number. The curve marked i_{gt} represents the total current when both alternators are running. In Fig. 13a, the curve marked i_{g1} represents the variable component of the grid current when the plate alternator is not running. The ratio of i_{g1} to e_g is, by definition, equal to the grid conductance. We may, therefore, write the equation

$$i_{g1} = G_g e_g = G_g 10 \sin \omega t \tag{8}$$

The curve marked i_{g2} represents the variable grid current when the grid alternator is not running. The ratio of i_{g2}

to e_p is by definition, equal to the controlled grid conductance. We may, therefore, write the equation

$$i_{g2} = G_{cg}e_p = G_{cg}100 \sin \omega t \tag{9}$$

The variable component of the grid current when both alternators are running is given by the equation

$$i_g = G_g e_g + G_{cg}e_p \tag{10}$$

Equations (7) and (10) apply rigorously only when the characteristic curves of the tube under consideration are straight lines. Operating points can generally be chosen so that Eq. (7) applies very closely. Equation (10), however, gives only an approximation to the actual variable grid current.

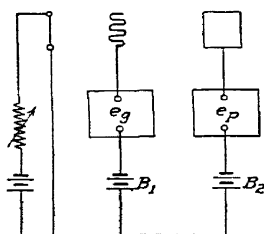



Fig. 13*b*

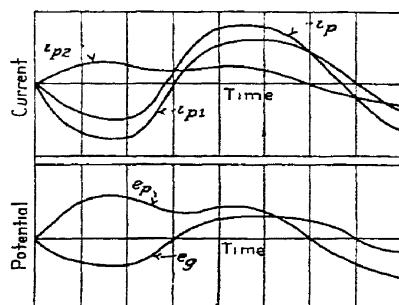



Fig 13*c*.—Variable components of plate potential, grid potential and plate current $G_{cp} = 4G_p$

Let the e.m.f. of the batteries B_1 and B_2 of Fig. 13*b* be adjusted so that the operating point lies in a region *A* of the characteristic surface over which a linear relation exists between the variable components of the plate current, the grid potential, and the plate potential. Let a variable potential e_g be applied to the grid while the plate potential is kept constant. This variable grid potential may be any function of the time provided only that its maximum variation is limited so that operation takes place only over the plane region *A* of the characteristic surface. The variable component of the grid potential is represented by the curve marked e_g in Fig. 13*c*.

Because of the manner in which the controlled plate conductance was defined, the variable component of the plate current is given by the relation

$$i_{p1} = G_{cp}e_g \tag{10a}$$

This current is represented in Fig. 13*c* by the curve marked i_{p1}.

Now, let the grid potential be maintained constant but let a variable potential represented by e_p be applied to the plate. This variable plate potential may be any function of the time provided only that its maximum variation is limited so that operation takes place only over the plane region *A* of the characteristic surface. The variable component of the plate potential is represented in Fig. 13*c* by the curve marked e_p. Because of the manner in which the plate conductance was defined, the variable component of the plate current is given by the relation

$$i_{p2} = G_p e_p \tag{10b}$$

This current is represented in Fig. 13*c* by the curve marked i_{p2}.

Now let the variable potential e_g be applied to the grid and the variable potential e_p be applied to the plate. Because linear relations exist between the plate current, the grid potential, and the plate potential over the region of the characteristic surface to which operation is confined, the variable plate current is given by the equation

$$i_p = i_{p1} + i_{p2} = G_{cp}e_g + G_p e_p \tag{11}$$

This current is represented by the curve marked i_p in Fig. 13*c*. The curve i_p is obtained by adding the curve i_{p1} to i_{p2}.

The variable grid current is given approximately by the equation

$$i_g = G_g e_g + G_{cg} e_p \tag{12}$$

Equations (11) and (12) are fundamental to the whole theory of triode circuits. Their application will be illustrated in the sections which follow.

6. The Simple Amplifier.

The elements of the simple amplifier circuit are shown by Fig. 14. The device supplying the power to be amplified is represented by the alternator A. The device in which the amplified power is to be utilized is represented by the resistance R_p in the plate circuit. The batteries D_1 and D_2 are adjusted so as to bring the tube to a desirable operating point. The questions to which we seek the answers are these:

1. What is the expression for the **power amplification?**
2. Under what condition do we obtain **maximum power amplification?**
3. What is the expression for the **maximum amplification?**
4. What is the expression for the **voltage amplification?**
5. Under what conditions do we obtain **maximum voltage amplification?**
6. What is the expression for the **maximum voltage amplification?**

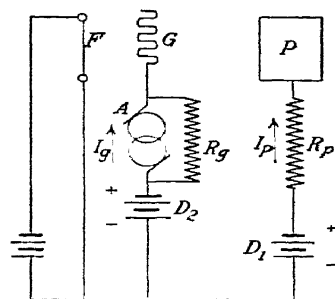



FIG 14.—Simple amplifier circuit

In asking these questions we have introduced the two new terms, **power amplification** and **voltage amplification.**

By **power amplification** is meant the ratio of that part of the power expended in the output element R_p which is controlled by the alternator A to the power supplied by the alternator.

By **voltage amplification** is meant the ratio of the alternating voltage across the output element R_p to the voltage of the alternator.

Before we can apply the theory developed in Sec. 5 to answer these questions, we must fix conditions so that operation takes place over characteristic curves which are essentially straight lines. Let the plate current-grid voltage characteristics be given by the curves of Fig. 6. Then if we choose some such point as o for an operating point and limit the variations in grid and plate potentials so that operation is confined to the space inclosed by the dotted

line *abcde*, we may use Eq. (11) to obtain the variable plate current. Let the grid current-grid voltage characteristic curves be given by Fig. 7. The operating point already chosen is +20 volts on the grid and +500 volts on the plate. Except for very small variations in potentials the curves are not straight lines in the vicinity of this point. We can remedy this condition in two ways: first, we can change the operating point so that the grid potential always remains negative, second, we can insert a resistance (R_g, Fig. 14) across the alternator terminals. The conductance of this resistance should be about ten times the grid conductance. It should be noted that the grid conductance and the resistance R_g are in parallel with respect to the alternator A.

We have now cleared the way so that the questions asked at the beginning of this section can be answered. In the treatment of amplifiers given in this chapter, the capacitances between triode elements are neglected. These capacitances are taken into account in the treatment of amplifiers given in Chap VIII, and the degree of approximation of the equations developed in the present chapter are discussed there. It can be pointed out, however, that if the external impedances are much smaller than the internal capacity reactances of the triode, then the equations about to be developed give a close approximation to actual conditions.

Let the terminal e.m.f. of the alternator be $\sqrt{2}E_g \sin \omega t$. Then the variable component of the grid potential is given by the relation

$$e_g = \sqrt{2}E_g \sin \omega t \tag{13}$$

The only variation which can take place in the plate potential is that due to the variable plate current flowing through the resistance R_p. The variable component of the plate potential is given by the relation

$$e_p = -R_p i_p \tag{14}$$

Upon substituting Eqs. (13) and (14) in Eq. (11), we obtain for the variable component of the plate current

$$i_p = \sqrt{2}G_{cp}E_g \sin \omega t - G_p R_p i_p \tag{15}$$

Solving for i_p, we obtain

$$i_p = \frac{\sqrt{2}G_{cp}E_g}{1 + R_pG_p} \sin \omega t \qquad (16)$$

Let the continuous plate current be represented by I_{pp}. Then the total plate current is

$$i_{pt} = I_{pp} + i_p = I_{pp} + \frac{\sqrt{2}G_{cp}E_g}{1 + R_pG_p} \sin \omega t \qquad (17)$$

Since this current must flow through the resistance R_p, the total average power expended in this resistance is

$$P_t = \frac{1}{T}\int_0^T R_p i_{pt}^2 dt = \frac{1}{T}\int_0^T R_p I_{pp}^2 dt + \frac{1}{T}\int_0^T \frac{2\sqrt{2}R_pI_{pp}G_{cp}E_g}{1 + R_pG_p} \sin \omega t dt + \frac{1}{T}\int_0^T \frac{2G_{cp}^2E_g^2R_p}{(1 + R_pG_p)^2} \sin^2 \omega t dt$$

$$P_t = R_pI_{pp}^2 + \frac{G_{cp}^2E_g^2R_p}{(1 + R_pG_p)^2} \qquad (18)$$

Of the power given by Eq. (18), the amount $R_pI_{pp}^2$ is expended whether or not the alternator is present. The second term of Eq. (18) must therefore give the power expended in the output resistance due to the presence of the alternator in the grid circuit. The power output may then be written as

$$P_0 = \frac{G_{cp}^2E_g^2R_p}{(1 + R_pG_p)^2} \qquad (19)$$

With the resistance around the grid alternator, the conductance between alternator terminals is equal to the actual grid conductance plus the conductance around the alternator. In the work which follows, G_g will represent the total conductance of the two parallel paths with respect to the alternator unless the contrary use of G_g is specifically stated to be used.

Upon substituting Eqs. (13) and (14) in Eq. (12), we obtain for the variable component of the grid current

$$i_g = \sqrt{2}G_gE_g \sin \omega t - G_{cg}i_pR_p \qquad (20)$$

Substituting Eq. (16) in Eq. (20) there results

$$i_g = \left[\frac{-\sqrt{2}G_{cp}E_gR_pG_{cg}}{1 + R_pG_p} + \sqrt{2}E_gG_g \right] \sin \omega t \qquad (21)$$

From the usual alternating-current theory, the power supplied by the alternator is equal to one-half of the product of the peak value of the current by the peak value of the potential. That is, the power input supplied by the alternator is given by the equation

$$P_i = E_g^2G_g - \frac{G_{cp}G_{cg}R_pE_g^2}{1 + R_pG_p} \qquad (21a)$$

The power amplification is given by the ratio

$$\frac{P_0}{P_i} = \frac{G_{cp}^2R_p}{(1 + R_pG_p)(G_g + G_gG_pR_p - G_{cp}G_{cg}R_p)} \qquad (22)$$

The value which the resistance R_p in the plate circuit must have in order to lead to maximum amplification of the power may be determined by taking the derivative of the amplification with respect to R_p, equating the derivative to zero, and solving the resulting equation for R_p. The value which R_p must have in order that the power amplification may be a maximum is found to be

R_p (For maximum power amplification) =

$$\frac{1}{G_p\sqrt{1 - \dfrac{G_{cp}}{G_p}\dfrac{G_{cg}}{G_g}}} \qquad (23)$$

Upon substituting the value of R_p as given by Eq. (23) in Eq. (22), we obtain the expression for maximum power amplification.

Maximum power amplification =

$$\frac{G_{cp}^2}{G_pG_g\left[1 + \sqrt{1 - \dfrac{G_{cp}}{G_p}\dfrac{G_{cg}}{G_g}}\right]^2} \qquad (24)$$

If we examine the table of triode constants, we find that the term $\left(\frac{G_{cp}}{G_p}\right)\left(\frac{G_{cg}}{G_g}\right)$ can in most cases be neglected in com-

parison with unity. Under these conditions the following very close approximations may be written:

$$R_p \text{ (For maximum power amplification)} = \frac{1}{G_p} \qquad (25)$$

$$\text{Maximum power amplification} = \frac{G_{cp}^2}{4G_gG_p} \qquad (26)$$

The voltage across the output resistance due to the presence of the alternator in the grid circuit is

$$\begin{aligned} e_p &= -R_pi_p \\ &= -\frac{\sqrt{2}E_gG_{cp}R_p}{1+R_pG_p}\sin\omega t \qquad (27) \end{aligned}$$

The alternator voltage is of course given by Eq. (13); so the voltage amplification is

$$\text{Voltage amplification} = \frac{e_p}{e_g} = \frac{G_{cp}R_p}{1+R_pG_p} = \frac{G_{cp}}{\frac{1}{R_p}+G_p} \qquad (28)$$

By inspection we see that maximum voltage amplification occurs when $R_p = \infty$. The value of this maximum voltage amplification is

$$\text{Maximum voltage amplification} = \frac{G_{cp}}{G_p} \qquad (29)$$

When the conditions for maximum power amplification are satisfied, R_p has the value given by Eq. (25). Substituting this value in Eq. (28), we find that the voltage amplification at maximum power amplification is

$$\text{Voltage amplification at maximum power amplification} = \frac{G_{cp}}{2G_p} \qquad (30)$$

That is, it is just one-half the maximum possible value. The maximum voltage amplification of a tube is represented by the symbol μ. This symbol is called the amplification constant of the tube. In our notation

$$\mu = \frac{G_{cp}}{G_p} \qquad (31)$$

That is, μ is the value of the voltage amplification with infinite impedance in the plate circuit.

We are now in a position to see what properties a triode should possess in order to be a good amplifier. From Eq. (26) we see that in order to obtain large amplification the ratios $\frac{G_{cp}}{G_p}$ and $\frac{G_{cp}}{G_g}$ should be as large as possible. The latter of these two ratios can usually be made large by operating the tube with such a large negative potential that the variations in grid potential never cause the grid to become positive. Therefore, the magnitude of the amplification constant μ is a good criterion of the value of a tube as a power and as a voltage amplifier.

In most communication work it is essential that the output of an amplifier be an exact magnification of the input. In the analytical work above, we have assumed the characteristic curves to be straight lines and we have found that a sine wave of input voltage gives a sine wave of current and voltage in the output circuit. Now any impressed wave form can be broken up into a series of sine and cosine terms and, if the triode has straight lines for characteristic curves, then each sine and cosine term in the input circuit would lead to a magnified sine or cosine term in the output circuit. The output would then be an exact magnification of the input. Thus for distortionless amplification a tube should have characteristic curves which are essentially straight lines over the operating region. Operating points for straight amplifiers should always be chosen so as to fulfil as nearly as possible this condition.

In order to obtain a curve for the power amplification of a triode which holds for all tubes as a function of the output resistance, let us set

$$R_p = k\frac{1}{G_p} \tag{26a}$$

If G_{cg} is taken equal to zero, the expression for the power amplification takes the form

$$A_p = \frac{G_{cp}^2 R_p}{G_g(1 + R_pG_p)^2} = \frac{G_{cp}^2 k}{G_pG_g(1 + k)^2} = \left(\frac{G_{cp}^2}{4G_gG_p}\right)\left(\frac{4k}{(1 + k)^2}\right)$$

By the use of Eq. (26), this becomes

$$A_p = \frac{4k}{(1 + k)^2} A_{pm} \tag{26b}$$

In the equation A_{pm} stands for the maximum power amplification of the triode. Curve 1 of Fig. 14*a* shows the

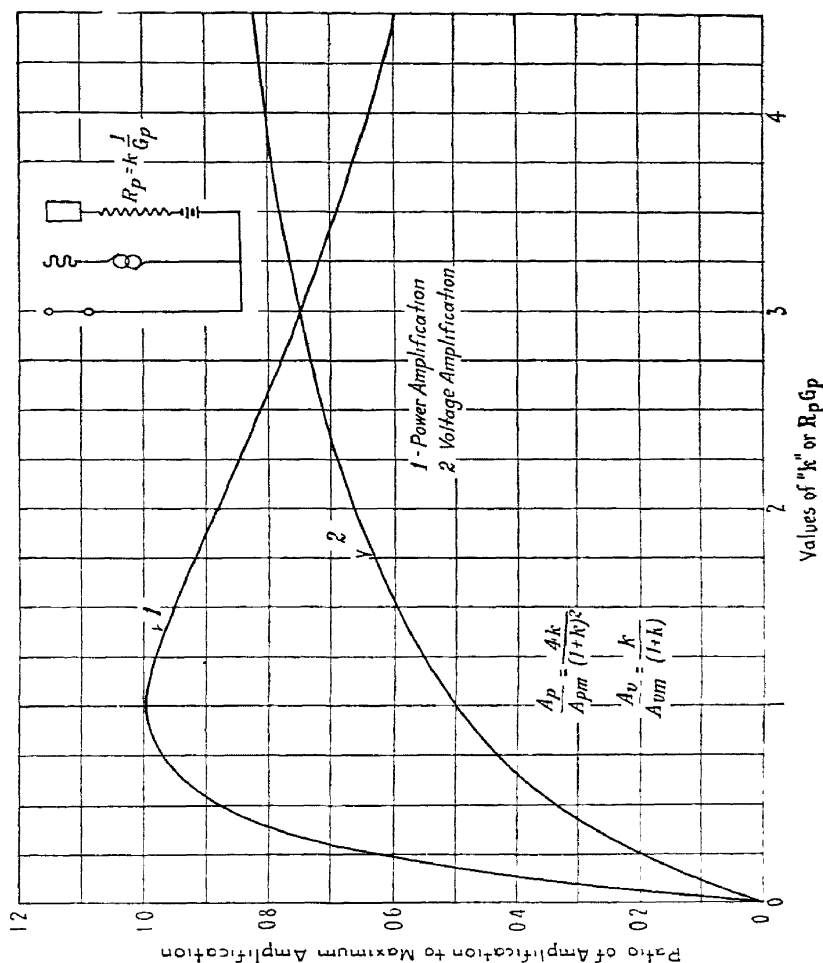



FIG 14*a*.—Amplification curves for simple amplifier

manner in which the power amplification varies with the output resistance. The ratio of power amplification to maximum power amplification (A_p to A_{pm}) has been plotted as ordinates, and values of k have been plotted as abscissas. This curve shows that as we increase the output resistance,

the power amplification rises rapidly to a maximum. This maximum occurs when the output resistance is equal to the reciprocal of the plate conductance. As the output resistance is increased beyond this point, the power amplification falls slowly to zero. The adjustment of the output resistance for maximum power amplification is not a critical one, as R_p may vary from $(0.5)\frac{1}{G_p}$ to $\frac{2}{G_p}$ without causing the power amplification to vary by more than 11 per cent from the maximum value. For a 201-A tube, $\frac{1}{G_p} =$ 10,800 ohms. The output resistance may therefore vary from 5,400 ohms to 21,600 ohms without causing the power amplification to differ by more than 11 per cent from the maximum value.

If R_p is again represented by Eq. (26*a*) and this value is substitued in Eq. (28), we obtain for voltage amplification the equation

$$A_v = \left(\frac{k}{1+k}\right)\frac{G_{cp}}{G_p} = \left(\frac{k}{1+k}\right)A_{vm} \tag{28a}$$

Curve 2 of Fig. 14*a* shows the manner in which the voltage amplification varies with the value of the output resistance. The voltage amplification approaches its maximum value asymptotically as the value of the output resistance is increased. If the voltage amplification is to be within 10 per cent of its maximum value, then

$$\frac{k}{1+k} \text{ must equal } 0.9$$

or k must equal 9; that is, the output resistance must equal nine times the reciprocal of the plate conductance. For the *radiotron* 201-A, R_p must equal $9 \times 10{,}800 = 97{,}200$ ohms if the voltage amplification is to be within 10 per cent of the maximum obtainable value. It is very seldom feasible to obtain values of voltage amplification much greater than 0.9 of the maximum possible value with a pure resistance in

the output because of the high voltage which this necessitates for the plate or B battery. The plate space current of the 201-A tube at the usual operating point (70 volts on plate) is about 3 milliamperes. The drop through a resistance of 97,200 ohms would be 292 volts so that to maintain 70 volts on the plate would require a battery voltage of 362 volts. To obtain a voltage amplification equal to 0.95 of the maximum value, R_p would have to be $19 \times \frac{1}{G_p}$. This would be a resistance of $19 \times 10{,}800 = 205{,}000$ ohms for a 201-A tube. To maintain 70 volts on the plate of the tube would require that the B battery voltage equal $(205{,}000)(0.003) + 70 = 685$ volts. In practice it is common to use about 100,000 ohms in the plate circuit of voltage amplifiers and to use a 130-volt B battery. This reduces the plate potential of a 201-A tube to about 36 volts.

7. Straight Amplifier Circuit in Which the Utilization Device is Not a Pure Resistance.

In many cases the utilization device, that is, the power receiving device in the plate circuit, contains both resistance and reactance. We therefore proceed to discuss the case in which the impedance in the output circuit has both resistance and reactance. The circuit under consideration is the same as the one shown by Fig. 14 with the one exception that R_p is replaced by a device having a resistance R_p and a reactance X_p.

Here, as in the case considered before, the potentials and currents consist of steady components upon which are superimposed sine waves of potential and current. The steady components and the alternating components can be treated separately and the final results obtained by superposition. Since we are primarily interested in the alternating components of potentials, currents, and powers, these components only will be treated. These alternating currents and potentials will be treated in the usual way by means of the complex algebra.

Let the terminal e.m.f. of the alternator in the grid circuit be taken as a reference vector. Let its r.m.s. vector

value be represented by **E**. Let the impedance in the plate circuit be represented by **Z**. That is,

$$\mathbf{Z} = R_p + jX_p \tag{32}$$

Then the alternating plate voltage is

$$\mathbf{E}_p = -\mathbf{Z}\mathbf{I}_p \tag{33}$$

The alternating plate space current is

$$\mathbf{I}_p = G_{cp}\mathbf{E} - \mathbf{I}_p\mathbf{Z}G_p \tag{34}$$

$$\mathbf{I}_p = \frac{\mathbf{E}G_{cp}}{1 + \mathbf{Z}G_p} \tag{35}$$

The alternating power expended in the utilization device is

$$P_0 = R_pI_p^2 = \frac{E^2G_{cp}^2R_p}{(1 + R_pG_p)^2 + X_p^2G_p^2} \tag{36}$$

If the controlled conductance of the grid by the plate is neglected, the expression for the grid current becomes

$$\mathbf{I}_g = \mathbf{E}G_g \tag{37}$$

The power furnished by the alternator is

$$P_i = E^2G_g \tag{38}$$

The expression for the power amplification then is

$$\frac{P_0}{P_i} = \frac{G_{cp}^2R_p}{G_g[(1 + R_pG_p)^2 + X_p^2G_p^2]} \tag{39}$$

Upon inspecting Eq. (39) and making use of the information gained in our study of the amplifier with a pure resistance in the output circuit, we see that the amplification will be a maximum when

$$X_p = 0\ ;\ R_p = \frac{1}{G_p} \tag{40}$$

When the reactance X_p is fixed at some value, the best value for R_p is found by taking the partial derivative of Eq. (39) with respect to R_p, equating this derivative to zero and solving for R_p. Upon carrying out these operations, we find that for maximum power amplification, R_p should have the value

$$R_p = \sqrt{X_p^2 + \frac{1}{G_p^2}} \tag{41}$$

One important case arises in which the utilization device can be designed so as to have a constant ratio of effective resistance to reactance. For this case $X_p = pR_p$. To find the value of R_p which will make the power amplification a maximum, we set $X_p = pR_p$ in Eq. (39) and take the partial derivative of Eq. (39) with respect to R_p, equate this derivative to zero, and solve for R_p. The best value for R_p is found to be

$$R_p = \frac{1}{G_p\sqrt{1 + p^2}} \tag{42}$$

Upon substituting Eq. (42) in Eq. (39), we obtain for the maximum power amplification:

$$\text{Maximum power amplification} = \frac{G^c{}_p}{2G_pG_g[1 + \sqrt{1 + p^2}]} \tag{43}$$

When there is reactance in the output circuit, the voltage amplification is found as follows: Substitute Eq. (35) in Eq. (33) and obtain the expression for the plate alternating voltage.

$$\mathbf{E}_p = \frac{\mathbf{E}ZG_{cp}}{1 + ZG_p} \tag{44}$$

The voltage amplification is

$$\frac{\mathbf{E}_p}{\mathbf{E}} = \frac{G_{cp}Z}{1 + ZG_p} = \frac{G_{cp}}{\frac{1}{Z} + G_p} \tag{45}$$

The voltage amplification is seen to be a maximum when $Z = \infty$. The maximum value of voltage amplification is again found to be

$$\text{Maximum voltage amplification} = \frac{G_{cp}}{G_p} = \mu \tag{46}$$

8. Multistage Amplification.

When the amplification which can be secured with one triode is not great enough to accomplish the desired magnification of the input power, the output of one tube is

connected to the input of a second and the output of the second to the input of a third and so on until as many tubes are used as is feasible. This is called multistage amplification.

There are a number of ways of connecting triodes so as to obtain multistage amplification. Most of these, however, are based upon two fundamental methods. We will proceed to discuss briefly these two methods. The first method may be called direct impedance coupling. A three-stage amplifier employing this coupling is shown schematically by Fig. 15. The power to be amplified is supplied by

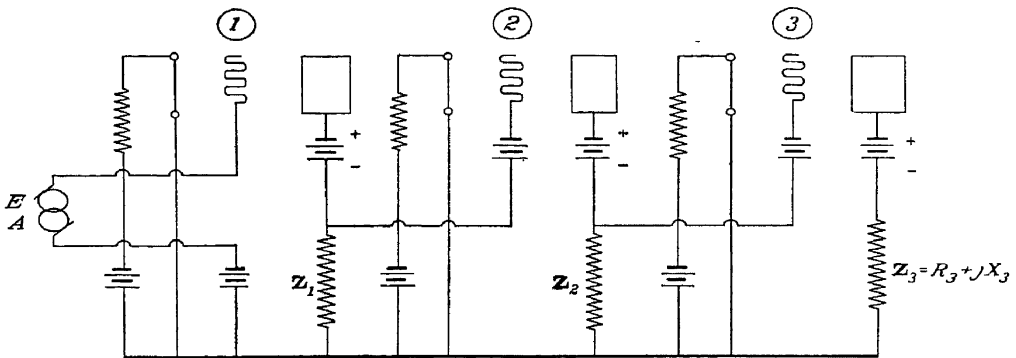



FIG 15.—Impedance coupled amplifier.

the alternator A; Z_1 and Z_2 are impedances built up in any manner whatsoever with the one requirement that they permit the passage of a direct current through them. Z_3 represents the impedance of the utilization device. It, also, must permit the passage of a direct current through it. The various batteries must be adjusted so that each tube comes to a desirable operating point. The amplification through the system is easily obtained from the equation developed in preceding sections.

The impedance in the first plate circuit is the impedance of Z_1 and the second grid conductance in parallel; that is,

$$Z_{p1} = \frac{1}{\frac{1}{Z_1} + G_{g2}}$$

The voltage amplification through the first tube is obtained from Eq. (45). Let this voltage amplification be represented by $\mathbf{A}_1$; that is,

$$\mathbf{A}_1 = \frac{G_{cp1}\mathbf{Z}_{p1}}{1 + \mathbf{Z}_{p1}G_{p1}} \tag{47}$$

The voltage on the second grid is

$$\mathbf{E}_{g2} = \mathbf{E}\mathbf{A}_1 \tag{48}$$

The impedance in the plate circuit of the second tube is

$$\mathbf{Z}_{p2} = \frac{1}{\dfrac{1}{\mathbf{Z}_2} + G_{g3}} \tag{49}$$

The voltage amplification through the second tube is

$$\mathbf{A}_2 = \frac{G_{cp2}\mathbf{Z}_{p2}}{1 + \mathbf{Z}_{p2}G_{p2}} \tag{50}$$

The voltage on the third grid is

$$\mathbf{E}_{g3} = \mathbf{A}_2\mathbf{E}_{g2} = \mathbf{A}_1\mathbf{A}_2\mathbf{E} \tag{51}$$

The power expended in the third grid space is

$$P_{i3} = E_{g3}^2G_{g3} = (A_1A_2E)^2G_{g3} \tag{52}$$

It should be recognized that the $\mathbf{A}_1$ and $\mathbf{A}_2$ defined by Eqs. (47) and (50) are complex numbers and that the A_1 and A_2 appearing in Eq. (52) and in all of the following power equations are the absolute values of the voltage amplifications as defined by Eqs. (47) and (50). From Eq. (39) the power amplification through the third tube is

$$A_3 = \frac{G_{cp3}^2R_3}{G_{g3}[(1 + R_3G_{p3})^2 + X_3^2G_{p3}^2]} \tag{53}$$

The power expended in the utilization device is

$$P_0 = P_{i3}A_3 = E^2A_1^2A_2^2A_3G_{g3} \tag{54}$$

The power furnished by the alternator is

$$P_i = E^2G_{g1} \tag{55}$$

The power amplification through the system is

$$\frac{P_0}{P_i} = A_1^2A_2^2A_3\frac{G_{g3}}{G_{g1}} \tag{56}$$

The maximum amplification is obtained in this system when Z_1, G_{g2}, Z_2, and G_{g3} are such that $\frac{1}{Z_{p1}}$ is small compared to G_{p1}, and $\frac{1}{Z_{p2}}$ is small compared to G_{p2}, and when $X_3 = 0$ and $R_3 = \frac{1}{G_{p3}}$. Under these conditions,

$$A_1 = \mu_1;\ A_2 = \mu_2;\ A_3 = \frac{G_{c\,p3}^2}{4G_{g3}G_{p3}}$$

The limiting value of the power amplification in a system such as Fig. 15 then is

$$\frac{P_0}{P_i}\ (\text{maximum}) = \mu_1^2\mu_2^2\,\frac{G_{c\,p3}^2}{4G_{p3}G_{g1}} \tag{57}$$

$$= \frac{G_{c\,p1}^2 G_{c\,p2}^2 G_{c\,p3}^2}{4G_{p1}^2 G_{p2}^2 G_{p3} G_{g1}} \tag{58}$$

The second method of connecting tubes so as to obtain multistage amplification is by means of impedance correct-

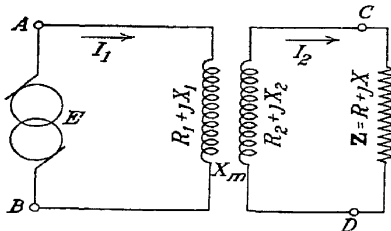



FIG 16.—Transformer circuit.

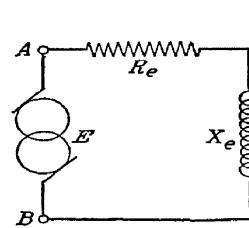



FIG 17.—Equivalent generator circuit

ing transformers. Before discussing this case, we will make a few remarks about the transformers. Consider the circuit shown by Fig. 16. Upon applying Kirchoff's law to the primary circuit we obtain

$$\mathbf{E} - R_1\mathbf{I}_1 - jX_1\mathbf{I}_1 - jX_m\mathbf{I}_2 = 0 \tag{59}$$

From the secondary circuit we obtain

$$-jX_m\mathbf{I}_1 - (R + R_2)\mathbf{I}_2 - j(X + X_2)\mathbf{I}_2 = 0 \tag{60}$$

$$I_2 = -\frac{jX_m\mathbf{I}_1}{(R + R_2) + j(X + X_2)} \tag{61}$$

Substituting Eq. (61) in Eq. (59) there results

$$\mathbf{E} - \mathbf{I}_1\left[R_1 + \frac{X_m^2(R + R_2)}{(R + R_2)^2 + (X + X_2)^2} + j\left\{X_1 - \frac{X_m^2(X + X_2)}{(R + R_2)^2 + (X + X_2)^2}\right\}\right] = 0 \qquad (62)$$

Now the term within the square brackets of Eq. (62) is the impedance of the system measured from the generator terminals A and B. So far as the generator is concerned, the system of Fig. 16 is replaceable by the one of Fig. 17 in which

$$R_e = R_1 + \frac{X_m^2(R + R_2)}{(R + R_2)^2 + (X + X_2)^2} \qquad (63)$$

$$X_e = X_1 - \frac{X_m^2(X + X_2)}{(R + R_2)^2 + (X + X_2)^2} \qquad (64)$$

Let the coefficient of magnetic coupling k between primary and secondary be defined by the equations:

$$M = k\sqrt{L_1L_2}$$

$$X_m = k\sqrt{X_1X_2} \qquad (65)$$

Substituting Eq. (65) in Eqs. (63) and (64), we have

$$R_e = R_1 + \frac{k^2X_1X_2\ (R + R_2)}{(R + R_2)^2 + (X + X_2)^2} \qquad (66)$$

$$X_e = X_1 - \frac{k^2X_1X_2(X + X_2)}{(R + R_2)^2 + (X + X_2)^2} \qquad (67)$$

Equations (66) and (67) are the general relations giving the effect on the impedance between a pair of terminals of inserting the transformers. Thus if we remove the transformers and join the terminals A to C and B to D, the impedance between the terminals AB is $\mathbf{Z} = R + jX$. Upon inserting the transformer, the impedance is given by Eqs. (66) and (67). We now lay down the following definition: An ideal transformer is one in which

1. The coefficient of coupling k is substantially equal to unity.

2. The open circuit reactance of the secondary winding is very great compared to any impedance which will ever be placed across it.

3. The resistance of the secondary winding is very small compared to R and that of the primary winding is small compared to R_e.

When these conditions are fulfilled, Eq. (66) simplifies as follows:

1. $k = 1$.

2. The denominator of the fraction becomes substantially equal to X_2.

3. R_2 can be dropped in comparison to R.

4. R_1 can be neglected in comparison to the fraction which follows it.

We then have

$$R_e = \frac{X_1 X_2}{X_2^2} R = \frac{X_1}{X_2} R \tag{68}$$

Equation (67) simplifies as follows:

$$X_e = X_1 - \frac{X_1 X_2 (X + X_2)}{(X + X_2)^2}$$
$$= \frac{X_1 X + X_1 X_2 - X_1 X_2}{X + X_2}$$

Thus we have very closely

$$X_e = \frac{X_1}{X_2} X \tag{69}$$

The ratio $\frac{X_1}{X_2}$ is called the ratio of impedance transformation from the secondary into the primary. It is equal to the square of the ratio of voltage transformation.

A two-stage transformer coupled multistage amplifier is shown schematically by Fig. 18. The device supplying the power to be amplified is the alternator A. This alternator has a resistance of R_a ohms and delivers a voltage whose r.m.s. value is E. The problem considered here is that of getting maximum power expenditure in R_p. The alternator A will deliver maximum power to the system when the apparent

resistance across its terminals is equal to R_a. From Eq. (68) we write

$$R_e = R_a = \frac{X_1}{X_2}\frac{1}{G_{g1}}$$

Or the transformer should have an impedance transformation ratio given by the relation

$$\frac{X_1}{X_2} = R_a G_{g1} \tag{70}$$

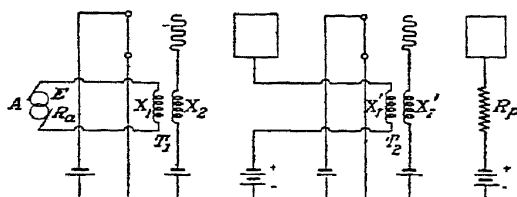



Fig 18.—Transformer coupled amplifier

For maximum power delivery to the second grid we know that the apparent resistance in the first plate circuit should be $\frac{1}{G_{p1}}$; so we write

$$\frac{1}{G_{p1}} = \frac{X'_1}{X'_2}\frac{1}{G_{g2}}$$

$$\frac{X^1}{X'_2} = \frac{G_{g2}}{G_{p1}} \tag{71}$$

For maximum power amplification through the second tube we have from Eq. (25)

$$R_p = \frac{1}{G_{p2}} \tag{72}$$

When the conditions of Eqs. (71) and (72) are fulfilled, the power amplification through the first tube is, from Eq. (26)

$$\frac{P_{01}}{P_{i1}} = \frac{G^2_{cp1}}{4G_{g1}G_{p1}}$$

The power output of the first tube is all expended in actuating the second grid so the power input to the second tube is

$$P_{i2} = P_{i1}\frac{G^2_{cp1}}{4G_{g1}G_{p1}}$$

The amplification through the second tube is

$$\frac{P_{02}}{P_{i2}} = \frac{G_{cp2}^2}{4G_{g2}G_{p2}}$$

The total amplification is

$$\frac{P_0}{P_i} = \left(\frac{G_{cp1}^2}{4G_{g1}G_{p1}}\right)\left(\frac{G_{cp2}^2}{4G_{g2}G_{p2}}\right) \tag{73}$$

If there are n tubes in the chain and if their amplifications are $A_1 \ . \ . \ . \ An$, the amplification through the chain when designed in accordance with Eqs. (71) and (72) is

$$\frac{P_0}{P_i} = (A_1)(A_2) \ . \ . \ . \ (A_n) \tag{74}$$

The amplification which can be obtained by the use of ideal or approximately ideal transformers is far in excess of that which can be obtained by straight impedance coupling.

If many stages of amplification are to be used, great care must be employed in the wiring, and each stage must be shielded by inclosing it in a metal-lined box. If this is not done, some energy from the output gets back into the input and is reamplified through the system. If the amplification is great enough, this leads to a sustained oscillation in the system commonly called **singing.**

9. Manner in Which Alternating Power Is Derived from Source of Continuous E.m.f.

In the treatment of amplification which has just been given, we found that alternating power was delivered to the utilization device in the plate circuit. Now in the plate or output circuit there was no source of alternating power; hence this alternating power must have been derived from the storage battery in the plate circuit. It is the purpose of this section to show just how this alternating power is obtained from the storage battery.

Consider again the simple amplifier circuit shown by Fig. 14. Before the alternator A in the grid circuit is started, the various currents and voltages are given as functions of time by the full lines of Fig. 19. The equations of the lines

representing the plate current and voltage before the alternator is started are

$$i_p = I_{pp} \tag{75}$$

$$e_p = E_{pp} \tag{76}$$

The power delivered by the plate battery is

$$P_{D1} = e_p i_p = E_{pp} I_{pp} \tag{77}$$

This power is expended in heating the plate.

After the alternator is started, the currents and voltages are given by the dotted lines of Fig. 19. The equations for the plate current and voltage now are

$$i_p = I_{pp} + \sqrt{2} I_p \sin \omega t \tag{78}$$

$$e_p = E_{pp} - \sqrt{2} E_p \sin \omega t \tag{79}$$

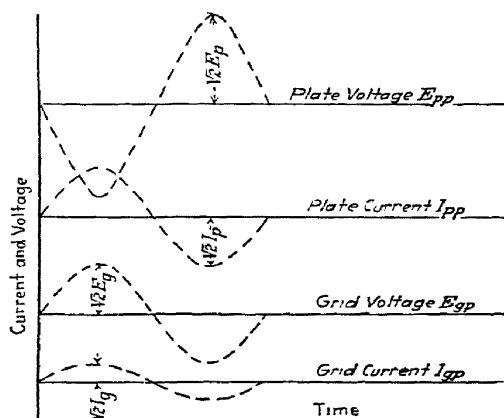



FIG. 19.—Potentials and currents in simple amplifier circuit.

The power expended in heating the plate now is

$$P_{Ht} = e_p i_p = E_{pp} I_{pp} - \sqrt{2} E_p I_{pp} \sin \omega t + \sqrt{2} E_{pp} I_p \sin \omega t - 2 E_p I_p \sin^2 \omega t \tag{80}$$

The average power consumed in heating the plate is

$$P_H = \frac{1}{T} \int_0^T e_p i_p dt = E_{pp} I_{pp} - E_p I_p \tag{81}$$

Equations (77) and (81) show that the power used in heating the plate after the alternator is started is less by the amount $E_p I_p$ than before the alternator was started. Now

the average current through the storage battery remains fixed at the value I_{pp} so the battery furnishes the power $E_{pp}I_{pp}$ both before and after the starting of the alternator. The amount of power E_pI_p is available for use in the output element. This is, of course, the amount of alternating power which we found was delivered to the output resistance R_p of Fig. 14 in our treatment of amplification. We thus see that the alternating power is drawn from the energy which would be used in heating the plate if the alternating currents and voltages were not present. When a large power tube ceases to deliver alternating power, the plates become very hot and may even melt unless precautions are taken to prevent it.

The physical reason why less energy is expended on the plate after starting the alternator than before, while the total average power delivered by the plate battery remains the same is this: The total charge transferred from the filament to the plate over any complete number of cycles is the same as the total charge transferred in the same interval of time before the alternator in the grid circuit was started. Before the alternator is started, all of this charge is transferred while the potential difference has the value E_{pp}. If the total charge is Q the energy expended on the plate before the alternator is started is QE_{pp}. Now the action of the grid is such that after the alternator is started, a greater part of Q is transferred from plate to filament when the potential difference is lower than normal than is transferred when the potential difference is higher than normal. This, of course, leads to less energy expenditure on the plate. Thus the action of the grid in holding back the charges when the plate potential is high and permitting the charge to pass when the plate potential is low accounts for the fact that alternating power is made available for use in the output or plate circuit.

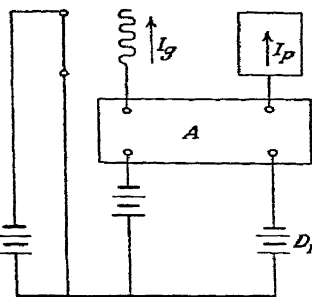
10. Conditions Which Must Be Fulfilled by Triode Circuits in Order to Obtain Alternating Power from Sources of Continuous E.m.f.

Let us now generalize our problem and ask what conditions in general must be fulfilled by triode circuits in order

that alternating power may be obtained from sources of continuous e.m.f. Figure 20 shows a general triode circuit. The box A contains auxiliary circuit elements, such as coils, condensers, resistances, and generators. The specific question asked here is this: What conditions must the apparatus in the box A fulfil in order that alternating power may be fed into it from the storage battery or direct-current generator D?

FIG. 20.

We consider here only the alternating components of the currents and voltages. These will be treated by the use of complex algebra.

Take the grid voltage as the reference vector. No matter in what manner this voltage arises, its representation in complex notation is $\mathbf{E}_g$, as it is the reference vector. No matter what gives rise to the plate voltage, it may be represented as

$$\mathbf{E}_p = E'_p + jE''_p \tag{82}$$

The alternating plate current then is

$$\begin{aligned}\mathbf{I}_p &= \mathbf{E}_g G_{cp} + \mathbf{E}_p G_p \\ &= E_g G_{cp} + E'_p G_p + jE''_p G_p\end{aligned} \tag{83}$$

Now it is to be particularly noted that, under our conventions, the plate voltage is the potential of the plate above the bus, or it is the potential impressed across the plate-filament space. Therefore the power product of plate current times plate voltage is the power expended in heating the plate. The alternating power expended in heating the plate, therefore, is

$$P_h = E'_p(E_g G_{cp} + E'_p G_p) + (E''_p)^2 G_p \tag{84}$$

Now, assuming straight-line characteristics for the triode, the power supplied by the battery D is the same whether or not there are alternating voltages on the plate and grid. If the battery then is to supply alternating power to the box A, less power must be expended in heating the plate after

the application of the alternating potentials than before; that is, the alternating power expended in heating the plate must be negative or there must be a power output instead of a power input. The power, then, as given by Eq. (84) must be negative. For this power to be negative E_p must be negative and

$$E'_pE_gG_{cp} > (E'_p)^2G_p + (E''_p)^2G_p \tag{85}$$

The physical meaning of these conditions is best obtained by referring to Fig. 21. Figure 21 is a general vector diagram for a triode when functioning so as to give an alternating power output. E_g represents the grid voltage. E'_p represents the real part of the plate voltage. We found

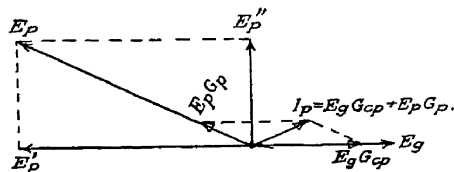



Fig. 21 —Vector diagram for a triode.

that E'_p must be negative; that is, it must be opposite in phase to E_g and it is so drawn on the diagram. E''_p may be either positive or negative. E_p, of course, represents the total plate voltage. The plate current as usual consists of two parts, one in phase with the plate voltage E_pG_p and one in phase with the grid voltage E_gG_{cp}. The total plate current is the vector sum of these two. For the tube to function as a generator the total plate current must be more than 90 degrees out of phase with the plate voltage. This condition is brought about by causing the plate and grid voltages to have components 180 degrees out of phase and by adjusting these voltages so that the plate current is controlled more by the grid than by the plate. We thus see that, due to the grid control of the plate current, the plate current can be made to increase while the impressed plate potential is caused to decrease. The plate space thus functions as a generator of alternating power.

As a simple example of the theory presented here, let the box A contain simply the two alternators as shown by Fig.

12. If power is to be fed into A_2, we must set the alternators A_1 and A_2 so that their voltages are out of phase with respect to the bus by more than 90 degrees. This fulfils the condition that E'_p should be negative. Now if the plate current is in phase with the grid voltage, it will be 180 degrees out of phase with the plate voltage, and the triode will function as a generator. To cause the plate current to follow the grid voltage rather than the plate voltage, we must satisfy the inequality

$$E_{A2}G_p < E_{A1}G_{cp}$$

$$\left(\frac{E_p}{E_g} = \frac{E_{A2}}{E_{A1}}\right) < \left(\frac{G_{cp}}{G_p} = \mu\right) \quad (86)$$

As approximate conditions for a triode to function as a generator we lay down the following:

1. The plate and grid voltages must be more than 90 degrees out of phase.

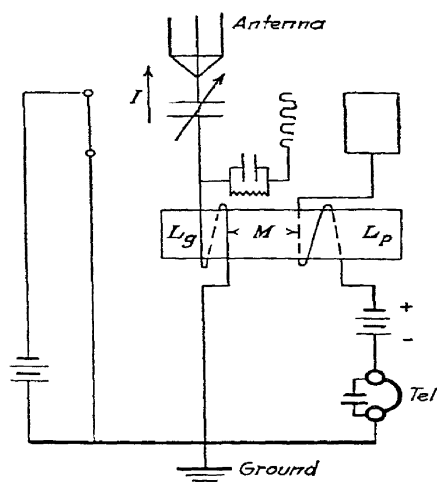
2. The ratio of the plate alternating voltage to the grid alternating voltage must be less than the voltage amplification constant of the tube.

Fig. 22

These conditions are approximate only in that the inequality to be satisfied, instead of being as simple as stated in condition 2 is given by Eq. (85). Equation (85) reduces to condition 2 when $E''_p = 0$, that is, when the grid and plate voltages are exactly 180 degrees out of phase.

As another example of the use of these generalized power relations, consider the radio receiving circuit shown by Fig. 22. If the circuit is highly oscillatory so that I is large compared to any other currents present, we have approximately

$$E_g = -j\omega L_g I$$
$$E_p = -j\omega M I$$

For E_p to be 180 degrees out of phase with E_g, M must be negative, or the coils must be wound as shown in the figure.

Also

$$\frac{E_p}{E_g} = \frac{j\omega MI}{j\omega L_g I} = \frac{M}{L_g} < \frac{G_{cp}}{G_p} \tag{87}$$

Thus for the tube to function as an amplifier, M must be negative, and the relation in Eq. (87) must be fulfiled.

Problems

3. Suppose that the characteristic curves of the triode in Fig. 12 are given by Fig. 6. Let the alternator in the grid circuit deliver the voltage $30 \sin \omega t$ and let the alternator in the plate circuit deliver the voltage $-200 \sin \omega t$. The reference point is as usual the common bus. Let the voltage of the grid battery be 20 and that of the plate battery be 500. Plot the total plate voltage and the total grid voltage. From the characteristic curves plot on the same sheet of paper the total plate current for one complete cycle of the alternators. Derive the plate current by a point-to-point method from the curves of the tube.

Does the plate alternator act as a generator or as a motor, and what power does it give out or absorb? State the arguments by which you arrive at the answer to the questions.

4. Given a simple amplifier circuit with a pure resistance in the plate branch such as the one shown by Fig. 14. The triode is a radiotron UV-201, whose constants are given in the table of triode constants. Plot curves with values of plate resistance as abscissa and values of power amplification and of voltage amplification as ordinates Is the adjustment for maximum power amplification a very critical one? About what relation must R_p bear to $\frac{1}{G_p}$ in order that the voltage amplification may be within 5 per cent of the maximum possible value?

5. Given a three-stage impedance-coupled amplifier such as the one shown by Fig. 15 The tubes are UV-201 radiotrons the constants of which are given in the table of triode constants What is the limiting value of the power amplification as Z_1, and Z_2 are made larger and larger and when Z_3 is assigned the best possible value? What is the best possible value of Z_3?

6. A two-stage impedance-coupled amplifier is to be designed to amplify a band of frequencies starting at 200 cycles and ending at 2,000 cycles. The coupling element consists of a pure reactance having the inductance L. If the power amplification over the above band of frequencies is not to vary by more than 2 per cent, what is the minimum value that can be assigned to L? The tubes are 201 radiotrons. The last tube has a pure resistance in the plate circuit.

7. Given a three-stage transformer-coupled amplifier such as the one shown by Fig. 18. The generator delivering the power to be amplified has a resistance of 2,000 ohms. The tubes are UV-201 radiotrons whose constants are given in the table of triode constants. Wanted the ratios of the transformers T_1, T_2, and T_3 and the value of the resistance R_p so that maximum power will be delivered to R_p. What is the value of the power amplification

through the system? How does this value compare with that obtained with the impedance-coupled amplifier?

8. Draw the circuit of Fig. 24 with the correct direction of plate winding so that the tube will feed power into the alternator circuit. Show by means of a vector diagram and the generalized power relations why you have drawn the winding as you have

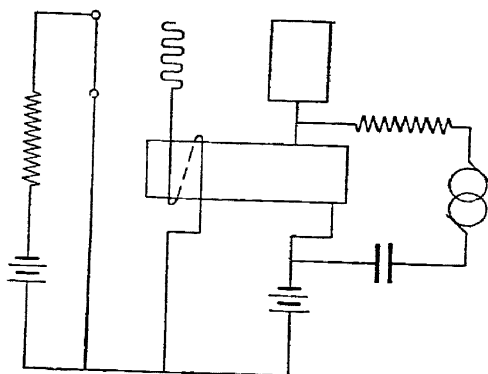



FIG 24.

CHAPTER III

RESISTANCE NEUTRALIZATION

11. Introduction to the Notion of Resistance Neutralization.

We now proceed to discuss those phenomena in triode circuits which are most strikingly described and most effectively treated as the phenomena of resistance neutralization. That is to say, the unique property of the circuits about to be studied is that the triode is associated with them in such a way as to **lower** the apparent **resistance** of some branch or branches of the network. In the mathematical

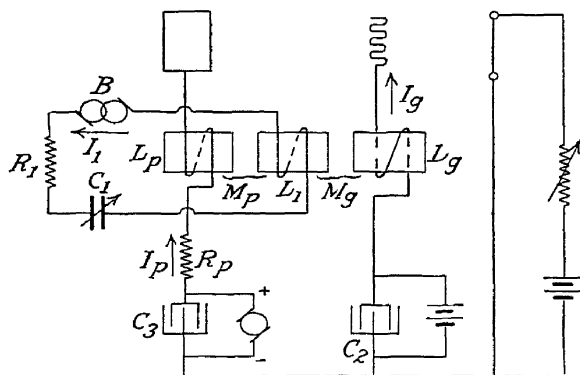



Fig. 25

treatment of these circuits, terms enter the equations which **subtract** from the resistance terms, and the resistance term may be made equal to zero or even may be made negative.

In this chapter the effect of resistance neutralization upon a simple series circuit will be worked out in detail, and the conclusions arrived at will then be used in the discussion of some triode circuit phenomena.

Before proceeding to a general treatment of resistance neutralization, we will first see how naturally we are led to describe certain triode circuit phenomena by assigning new

constants to a circuit associated with the triode. This will be done by a preliminary treatment of the circuit shown in Fig. 25. In treating this circuit it will be assumed that the grid is operated at such a continuous potential that $G_{cg} = G_g = 0$; that is, we assume that the grid current is negligibly small. Only the alternating components of currents and voltages will be considered, and these will be represented in the usual way by means of complex numbers.

Upon applying Kirchoff's e.m.f. law to circuit 1, we obtain

$$\mathbf{E} - R_1\mathbf{I}_1 - j\omega L_1\mathbf{I}_1 + \frac{j\mathbf{I}_1}{\omega C_1} - j\omega M_p\mathbf{I}_p = 0 \qquad (1)$$

The alternating plate voltage is

$$\mathbf{E}_p = -j(\omega M_p\mathbf{I}_1 + \omega L_p\mathbf{I}_p) - R_p\mathbf{I}_p \qquad (2)$$

The alternating grid voltage is

$$\mathbf{E}_g = -j\omega M_g\mathbf{I}_1 \qquad (3)$$

The alternating plate current is

$$\mathbf{I}_p = \mathbf{E}_g G_{cp} + \mathbf{E}_p G_p \qquad (4)$$

Upon substituting Eqs. (2) and (3) in Eq. (4), we obtain

$$\mathbf{I}_p = j\omega[-M_g G_{cp}\mathbf{I}_1 - (M_p\mathbf{I}_1 + L_p\mathbf{I}_p)G_p] - R_p G_p\mathbf{I}_p \qquad (5)$$

$$\mathbf{I}_p = \frac{j\omega[-M_g G_{cp} - M_p G_p]}{1 + R_p G_p + j\omega L_p G_p}\mathbf{I}_1 \qquad (6)$$

Let D represent

$$1 + R_p G_p \qquad (7)$$

and X_1 represent

$$\omega L_1 - \frac{1}{\omega C_1} \qquad (8)$$

Upon substituting Eqs. (6), (7), and (8) in Eq. (1) there results

$$\mathbf{E} - \mathbf{I}_1\left\{R_1 + jX_1 - \frac{\omega^2 M_p(-M_g G_{cp} - M_p G_p}{D + j\omega L_p G_p}\right\} = 0 \qquad (9)$$

Upon rationalizing Eq. (9), we obtain

$$\mathbf{E} - \mathbf{I}_1\left\{R_1 - \frac{\omega^2 hD}{D^2 + \omega^2 L_p^2 G_p^2} + j\left(X_1 + \frac{\omega^3 L_p G_p h}{D^2 + \omega^2 L_p^2 G_p^2}\right)\right\} = 0 \qquad (10)$$

Where h represents

$$M_p(-M_gG_{cp} - M_pG_p) \tag{11}$$

Equation (10) is of the form

$$\mathbf{E} - \mathbf{IZ} = 0$$

The effective impedance of circuit 1 after it is associated with the triode is thus seen to be the bracketed term of Eq. (10). Associating the triode with circuit 1 in the manner shown by Fig. 25 changes the steady-state resistance from

$$R_1 \text{ to } R_1 - \frac{\omega^2 hD}{D^2 + \omega^2 L_p^2 G_p^2}$$

and its steady-state reactance from

$$X_1 \text{ to } X_1 + \frac{\omega^3 L_p G_p h}{D^2 + \omega^2 L_p^2 G_p^2}$$

It is evident that the effective resistance of the circuit will be lowered if h is a positive quantity. Now h is a positive quantity if M_p and M_g have opposite signs and if $|M_gG_{cp}|$ is greater than $|M_pG_p|$. That is, the coupling must be as shown in Fig. 25 and the circuits so proportioned that $|M_gG_{cp}| > |M_pG_p|$ if the triode is to lower the effective resistance of circuit 1. These conditions are identical with the conditions laid down in Sec. 10, Chap. II, as necessary for the triode to feed power into circuit 1. We can see this in the following way: If the current I_1 is large compared to the plate space current, then from Eqs. (2) and (3), $E_p = -j\omega M_pI_1$ and $E_g = -j\omega M_gI_1$. These voltages will be 180 degrees out of phase if M_g and M_p have opposite signs. Also

$$\left|\frac{E_p}{E_g}\right| = \left|\frac{M_p}{M_g}\right|,$$

and if

$$|M_gG_{cp}| > |M_pG_p|,$$

then

$$|E_gG_{cp}| > |E_pG_p|.$$

Now we can describe the action of the triode on circuit 1 of Fig. 25 by stating that it lowers the effective resistance

and changes the reactance by the amounts shown by Eq. (10) provided we understand just what this change in resistance and reactance means. The effect of a change in reactance on a circuit is for the most part familiar, but the effect of lowering the resistance of a circuit is not so well known. After discussing the complete solution of the differential equation for the circuit of Fig. 25, the general theory of resistance neutralization will be developed so that it may be used in future discussions of triode circuits.

11a. Complete Equations for Fig. 25.

It is shown in Appendix B that if the triode circuit is designed to keep the reactive term low, a very close complete solution of the differential equations of the system is given by the following equations:

The current in circuit 1 is

$$i_1 = \frac{E}{\sqrt{(R_1 - \omega^2 h)^2 + X_n^2}} \cos (\omega t - \tau - \lambda) + \left[I_d \cos \beta t + \left(C_1 \beta E_{cd} + \frac{a}{\beta} I_d \right) \sin \beta t \right] \epsilon^{-\frac{R_1 - \beta^2 h}{2L_1} t} \qquad (12)$$

The counter electromotive force of the condenser C_1 is

$$e_c = \frac{E}{\omega C_1 \sqrt{(R_1 - \omega^2 h)^2 + X_n^2}} \cos \left(\omega t + \frac{\pi}{2} - \tau - \lambda \right) + \left[E_{cd} \cos \beta t - \left(\frac{E_{cd} a}{\beta} + \frac{I_d}{C_1 \beta} \right) \sin \beta t \right] \epsilon^{-\frac{R_1 - \beta^2 h}{2L_1} t} \qquad (13)$$

In these equations the symbols have the following meaning:

The alternating voltage impressed in circuit 1 is expressed by the equation $e = E \cos (\omega t - \tau)$, in which time is measured from the instant of switching in the voltage.

τ is the interval in radians from the moment of switching to the first positive peak of the impressed e.m.f.

λ is the angle of lag of the permanent current behind the impressed e.m.f. $= \tan^{-1} \dfrac{X_n}{R_1 - \omega^2 h}$

X_n is the net reactance $= \omega L_1 - \dfrac{1}{\omega C_1} + \omega^3 L_p G_p h$

$$I_d = I_o - \frac{E}{Z}\cos(\tau + \lambda)$$

$$E_{cd} = E_{co} - \frac{EX_c}{Z}\cos\left(\tau + \lambda - \frac{\pi}{2}\right)$$

Z is the net impedance $= \sqrt{(R_1 - \omega^2 h)^2 + X_n^2}$

$$X_c = \frac{1}{\omega C_1}$$

$$\Omega_r = \frac{1}{\sqrt{L_1 C_1}}$$

$$\beta = \Omega_r \sqrt{\frac{DL_1}{DL_1 + R_1 L_p G_p}}$$

$$a = -\frac{R_1 - \beta^2 h}{2L_1}$$

These equations are identical in form with the equations for the start of an alternating current in the circuit with the neutralizer omitted. The only difference is that for the circuit without the neutralizer, R must be substituted for $R_1 - \omega^2 h$ and $R_1 - \beta^2 h$, X_1 substituted for X_n, and $\dfrac{1}{\beta L_1}$ written for $C_1 \beta$. This latter substitution is legitimate if β differs little from Ω_r.

12. Conditions Necessary for Resistance Neutralization. Consider the series circuit shown by Fig. 26. Let B represent a device feeding power into the circuit. B may have any voltage characteristic whatsoever. Let A be a device which introduces into the circuit a voltage which is directly proportional to and in phase with the current in the circuit ($e_A = Ni_1; N > 0$). Then the device A produces an effect which will be called **pure resistance neutralization.** The effect is termed pure resistance neutralization because if the resistance R_1 and the device A were enclosed in a box D with two terminals e and f brought out, the box would act in all respects like a resistance of magnitude

$R_1 - N$. The truth of this proposition is indicated by the following general argument. When a current flows through R_1, a voltage arises across R_1 which is directly proportional to the current but in phase opposition to it. A introduces into the circuit a voltage proportional to the current but in phase with it. A thus acts just the opposite of R_1, and the box D should act as a resistance of magnitude $R_1 - N$.

The proof of the proposition is very simple. It consists of the comparison of the differential equation of the circuit of Fig. 26 with the differential equation of the same circuit with A not present. If the voltage of A is given by the relation

FIG 26.

$$e_A = Ni_1 \tag{14}$$

then Kirchoff's voltage law applied to the circuit of Fig. 26 gives the equation

$$e_B - L\frac{di_1}{dt} - i_1(R_1 - N) - \frac{q_1}{C} = 0 \tag{15}$$

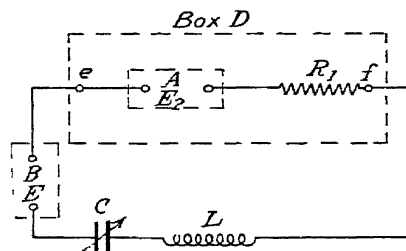
When A is not present in the circuit, the differential equation is

$$e_B - L\frac{di_1}{dt} - i_1R_1 - \frac{q_1}{C} = 0 \tag{16}$$

Equation (15) is sufficient to portray the relations in the circuit of Fig. 26 under all conditions, and Eq. (16) is sufficient to portray the relations in this same circuit when A is not present. Now Eq. (15) is the same as Eq. (16) if we replace R_1 in Eq. (16) by $R_1 - N$. That is, any solution of Eq. (16) becomes a solution of Eq. (15) if R_1 is replaced by $R_1 - N$. Therefore under all conditions and for all types of applied voltages, the box D of Fig. 26 acts as a resistance of magnitude $R_1 - N$.

13. Power Relations.

In the circuit of Fig. 26 let the device B represent an alternator delivering a sine electromotive force whose root-

mean-square value is E. Let X_n represent the net reactance of the circuit. Then, in the steady state, the current flowing in the circuit is

$$\mathbf{I} = \frac{\mathbf{E}}{(R - N) + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{\mathbf{E}}{(R - N) + jX_n} \tag{17}$$

$$I = \frac{E}{\sqrt{(R - N)^2 + X_n^2}} \tag{18}$$

If θ represents the phase angle between the alternator voltage and the current in the circuit, then the power delivered by the alternator B is

$$P_b = EI \cos \theta \tag{19}$$

$$\cos \theta = \frac{R - N}{\sqrt{(R - N)^2 + X_n^2}} \tag{20}$$

$$P_b = \frac{(R - N)E^2}{(R - N)^2 + X_n^2} \tag{21}$$

If the voltage of the neutralizer is $\mathbf{E}_a = N\mathbf{I}$, then since this voltage is in phase with the current, the power furnished by the neutralizer is

$$P_a = E_a \mathbf{I} = NI^2 \tag{22}$$

Equation (22) is the general equation for the power which must be furnished by a device which lowers the resistance of a circuit by the amount N. Upon substituting the value of I from Eq. (18) in Eq. (22), we obtain for the power furnished by the neutralizer

$$P_a = \frac{NE^2}{(R - N)^2 + X_n^2} \tag{23}$$

The total power delivered to the resistance is

$$P_t = RI^2 = \frac{RE^2}{(R - N)^2 + X_n^2} \tag{24}$$

It will be seen that $P_t = P_a + P_b$ as it should.

If the resistance neutralizer is not present in the circuit, the power delivered by the alternator is

$$P_0 = \frac{RE^2}{R^2 + X_n^2} \tag{25}$$

Let the **regenerative amplification** due to a resistance neutralizer be defined to be the ratio of the total power delivered to the circuit with the neutralizer present to the total power delivered to the circuit without the neutralizer. The regenerative amplification then is seen to be

$$\text{regenerative amplification} = \frac{P_t}{P_0} = \frac{R^2 + X_n^2}{(R - N)^2 + X_n^2} \quad (26)$$

If the circuit is resonant to the frequency of the alternator voltage, the expression for the regenerative amplification becomes

$$\text{regenerative amplification at resonance} = \frac{P_t}{P_0} = \frac{R^2}{(R - N)^2} \quad (27)$$

Let the ratio $\frac{R - N}{R}$ be represented by γ (28)

This ratio γ will be called the **reduction factor** of the neutralizer when associated with the particular circuit. It is the factor by which the total resistance of the circuit must be multiplied in order to obtain the reduced or net resistance. In terms of the reduction factor the regenerative amplification at resonance is

$$\text{regenerative amplification at resonance} = \left(\frac{1}{\gamma}\right)^2 \quad (29)$$

If the circuit is dissonant to the alternator frequency to such a degree that X_n is large compared to either R or $R - N$, then the regenerative amplification becomes

$$\text{regenerative amplification off resonance} = \frac{X_n^2}{X_n^2} = 1 \quad (30)$$

A comparison of Eqs. (29) and (30) brings out in a striking way the selective amplification due to a resistance neutralizer. This property of a resistance neutralizer has an important application in radio reception and will be taken up in detail in connection with the selective properties of circuits associated with resistance neutralizers.

Some important power ratios are as follows:

$$\frac{P_a}{P_b} = \frac{N}{R - N} = \frac{1}{\gamma} - 1 \tag{31}$$

$$\frac{P_t}{P_b} = \frac{R}{R - N} = \frac{1}{\gamma} \tag{32}$$

$$\frac{P_b}{P_0} = \left(\frac{R - N}{R}\right)\frac{R^2 + X_n^2}{(R - N)^2 + X_n^2} \tag{33}$$

$$\frac{P_a}{P_0} = \frac{N}{R}\frac{R^2 + X_n^2}{(R - N)^2 + X_n^2} \tag{34}$$

$$\frac{P_t}{P_0} = \frac{R^2 + X_n^2}{(R - N)^2 + X_n^2} \tag{35}$$

For the case in which the circuit is resonant to the alternator frequency, these power ratios reduce to the following forms. These ratios also apply to the direct-current case.

$$\frac{P_a}{P_b} = \frac{N}{R - N} = \frac{1}{\gamma} - 1 \tag{36}$$

$$\frac{P_t}{P_b} = \frac{R}{R - N} = \frac{1}{\gamma} \tag{37}$$

$$\frac{P_b}{P_0} = \frac{R}{R - N} = \frac{1}{\gamma} \tag{38}$$

$$\frac{P_a}{P_0} = \left(\frac{R}{R - N}\right)\left(\frac{N}{R - N}\right) = \frac{1}{\gamma}\left(\frac{1}{\gamma} - 1\right) = \left(\frac{1}{\gamma}\right)^2 - \frac{1}{\gamma} \tag{39}$$

$$\frac{P_t}{P_0} = \frac{R^2}{(R - N)^2} = \left(\frac{1}{\gamma}\right)^2 \tag{40}$$

Equation (31) shows that if γ is less than 0.5, the neutralizer must furnish more power than the generator, while if γ is greater than 0.5, the generator furnishes more power than the neutralizer. In good radio circuits γ is always much less than 0.5. Hence in these circuits the triode furnishes by far the greater share of the power.

If the circuit is resonant to the impressed frequency, Eq. (38) shows that the neutralizer causes the generator to deliver more power than it would if the neutralizer were

not present. If the circuit is so much detuned that X_n is large compared to R, then Eq. (33) becomes

$$\frac{P_b}{P_0} \text{(for off resonance)} = \frac{R - N}{R} = \gamma \quad (41)$$

That is, the neutralizer causes the generator to deliver **less power** than it would deliver if the neutralizer were not present. If Fig. 26 is the antenna circuit of a radio receiving set, then the voltage of B is the voltage induced in the antenna by impinging electromagnetic waves, and we see that the neutralizer causes the waves to deliver **more power at resonance** and **less power off resonance** than these waves would deliver if the neutralizer were not present.

14. Departures from Pure Resistance Neutralization.

If the device A introduces into the circuit of Fig. 26 a steady-state voltage given by the relation

$$\mathbf{E}_a = N\mathbf{I}_1 - jX_a\mathbf{I}_1 \quad (42)$$

then it is evident that, in the steady state, the box D will act as a resistance of magnitude $R_1 - N$ in series with a reactance of magnitude X_a. All of the equations which have been developed will still hold if X_n is understood to include X_a. That is, if X_n is given by the relation

$$X_n = \omega L - \frac{1}{\omega C} + X_a \quad (43)$$

X_a may be either a positive or a negative quantity. If the circuit were resonant to the impressed frequency before the introduction of the neutralizer, it will have to be retuned after the introduction of the neutralizer if resonance is still desired. A great majority of the triode circuits when used as neutralizers introduce in the circuits voltages as expressed in Eq. 42. Thus for the circuit of Fig. 25 we see from Eq. (10) of Sec. 11 that

$$N = \frac{\omega^2 hD}{D^2 + \omega^2 L_p^2 G_p^2} \quad (44)$$

$$X_a = \frac{\omega^3 L_p G_p h}{D^2 + \omega^2 L_p^2 G_p^2} \quad (45)$$

In order to gain some idea of the relative magnitude of these terms the constants of an experimental set-up assembled from ordinary laboratory coils and condensers will be given. For this particular set-up which is the circuit of Fig. 25 the constants were as follows:

$$\begin{aligned}
R_1 &= 80 \text{ ohms} \\
G_p &= 200 \times 10^{-6} \text{ mhos} \\
G_{cp} &= 3{,}000 \times 10^{-6} \text{ mhos} \\
R_p &= 5.13 \text{ ohms} \\
L_p &= 3{,}350 \times 10^{-6} \text{ henrys} \\
L_g &= 172 \times 10^{-6} \text{ henrys} \\
L_1 &= 20{,}320 \times 10^{-6} \text{ henrys} \\
M_p &= 3{,}000 \times 10^{-6} \text{ henrys} \\
M_g &= 432 \times 10^{-6} \text{ henrys} \\
C_1 &= 1.4 \times 10^{-9} \text{ farads (antenna)} \\
\omega_r &= 1.875 \times 10^5 \text{ radians per second} \\
R_g &= 1.16 \text{ ohms} \\
\omega_r^2 &= 3.515 \times 10^{10}
\end{aligned}$$

For this circuit we then find that

$$\begin{aligned}
D &= 1.00102 \\
\omega^2 L_p^2 G_p^2 &= 0.0158 \\
h &= 2.1 \times 10^{-9}
\end{aligned}$$

For this circuit then $D^2 + \omega^2 L_p^2 G_p^2$ may for most purposes be taken equal to unity. This will be done in the following equations:

$$\begin{aligned}
N &= \omega^2 h = 74 \text{ ohms} \\
X_a &= \omega^3 L_p G_p h = +9.23 \text{ ohms.}
\end{aligned}$$

15. Conditions Necessary for Triode Circuits to Function as a Resistance Neutralizer.

Since a resistance neutralizer must furnish an amount of power equal to NI_1^2, it is evident that one set of conditions which the triode circuits must fulfil in order to function as a resistance neutralizer is the same as those necessary for power output. These conditions were developed

and discussed in Sec. 10 of Chap. II. They are restated here in order to bring together in one section the conditions necessary for resistance neutralization. The conditions necessary for the tube to make power available in the output circuit are as follows:

1. The grid and plate variable potentials must have components 180 degrees out of phase.

2. If the root-mean-square value of the grid voltage is E_g and if the component of the plate voltage which is 180 degrees out of phase with the grid voltage is E'_p and the component of the plate voltage 90 degrees out of phase with the grid voltage is E''_p, then

$$E'_p E_g G_{cp} \text{ must be greater than } (E'_p)^2 G_p + (E''_p)^2 G_p \qquad (46)$$

If, as is generally the case, E''_p is small compared to E'_p this condition simplifies to

$$\frac{E_p}{E_g} \text{ must be less than } \frac{G_{cp}}{G_p} \qquad (47)$$

These conditions, however, are not sufficient for resistance neutralization because the power fed into the circuit whose resistance is to be neutralized must vary directly as the square of the current in that circuit. For convenience in reference let the circuit in which resistance is to be neutralized be designated as circuit 1. Now the resistance-neutralizing voltage introduced into circuit 1 by the triode must be controlled by the plate space current because this is the current with which the output power of the triode is associated. Since this resistance-neutralizing voltage must be proportional to the current in circuit 1, the plate space current must be controlled by the current in circuit 1. This requires that in the last analysis the grid and plate alternating potentials shall be controlled by the current in circuit 1.

Let us put these conditions in mathematical form. Let the resistance-neutralizing voltage be designated by E_a. Then if this voltage is controlled by the plate space current, we may write

$$\mathbf{E}_a = (p_1 + jp_2)\mathbf{I}_p \qquad (48)$$

where p_1 and p_2 are scalar constants depending upon the circuits used. If the grid and plate voltages are controlled by the current in circuit 1, we may write

$$\mathbf{E}_g = (V_1 + jV_2)\mathbf{I}_1 \tag{49}$$
$$\mathbf{E}_p = (U_1 + jU_2)\mathbf{I}_1 \tag{50}$$

where V_1, V_2, U_1, and U_2 are real constants depending upon the circuits under consideration. But

$$\begin{aligned}\mathbf{I}_p &= \mathbf{E}_g G_{cp} + \mathbf{E}_p G_p \\ &= \mathbf{I}_1[(V_1 + jV_2)G_{cp} + (U_1 + jU_2)G_p]\end{aligned} \tag{51}$$

Upon substituting Eq. (51) in Eq. (48), there results

$$\mathbf{E}_a = (p_1 + jp_2)[(V_1 + jV_2)G_{cp} + (U_1 + jU_2)G_p]\mathbf{I}_1 \tag{52}$$
$$\begin{aligned}&= \mathbf{I}_1\{[(p_1V_1 - p_2V_2)G_{cp} + (p_1U_1 - p_2U_2)G_p] \\ &\quad + j[(p_1V_2 + p_2V_1)G_{cp} + (p_1U_2 + p_2U_1)G_p]\}\end{aligned} \tag{53}$$

Equation (53) is the same form as Eq. (42), and upon comparing these two equations we write

$$N = [(p_1V_1 - p_2V_2)G_{cp} + (p_1U_1 - p_2U_2)G_p] \tag{54}$$
$$X_a = -[(p_1V_2 + p_2V_1)G_{cp} + (p_1U_2 + p_2U_1)G_p] \tag{55}$$

If the triode functions so as to feed power into circuit 1, N as given by Eq. (54) must be positive, and the conditions stated are sufficient to insure that the triode functions as a resistance neutralizer.

In order to illustrate the equations which have been used in this section, we shall apply them to the circuit of Fig. 25. From Eq. 1, Sec. 11, we see that the voltage introduced into circuit 1 by the plate space current is

$$\mathbf{E}_a = -j\omega M_p\mathbf{I}_p$$

Upon comparing this with Eq. (48), we write

$$p_1 + jp_2 = -j\omega M_p \tag{56}$$
$$p_1 = 0;\ p_2 = -\omega M_p \tag{57}$$

Upon comparing Eq. (49) with Eq. (3) of Sec. (11) we write

$$\begin{aligned}V_1 + jV_2 &= -j\omega M_g \\ V_1 = 0;\ V_2 &= -\omega M_g\end{aligned} \tag{58}$$

To obtain the values of U_1 and U_2 for this circuit, we must write the plate voltage in terms of the constants of the circuit and the oscillating current I_1. Upon substituting the value of I_p from Eq. (6) in Eq. (2) of Sec. 11, rationalizing and collecting term, we obtain

$$\mathbf{E}_p = \left\{ -\frac{R_p\omega^2hL_pG_p - \omega^2hL_pD}{(D^2 + \omega^2L_p^2G_p^2)M_p} - j\left[\omega M_p + \frac{\omega hR_pD + \omega^3hL_p^2G_p}{(D^2 + \omega^2L_p^2G_p^2)M_p}\right]\right\}\mathbf{I}_1 \quad (59)$$

Upon comparing Eq. (59) with Eq. (50) we write

$$U_1 = -\frac{R_p\omega^2hL_pG_p - \omega^2hL_pD}{(D^2 + \omega^2L_p^2G_p^2)M_p} \quad (60)$$

$$U_2 = -\left[\frac{\omega hR_pD + \omega^3hL_p^2G_p}{(D^2 + \omega^2L_p^2G_p^2)M_p} + \omega M_p\right] \quad (61)$$

If the values of U, V, and p as given by the above equations are substituted in Eq. (54) we obtain the expression for N as given by Eq. (44).

16. Expression for N Obtained from the Power Relation. In a great many of the circuits which fulfil the conditions for resistance neutralization, the plate and grid voltages are substantially 180 degrees out of phase and nearly all of the power output of the triode is used in neutralizing the resistance of the oscillating system of circuits associated with the triode. For these circuits it is possible to derive a very simple expression for N. This expression for N will be useful in many discussions, and from it the approximate value of N for a great many circuits can be written down in terms of the circuit constants.

We have shown that the plate space current is given by the relation

$$\mathbf{I}_p = \mathbf{E}_g G_{cp} + \mathbf{E}_p G_p$$

If the grid voltage is taken as the reference vector and if the plate voltage is 180 degrees out of phase with the grid voltage, the above expression can be written as

$$I_p = E_g G_{cp} - E_p G_p \quad (62)$$

The alternating power output is

$$P = E_pI_p = E_p(E_gG_{cp} - E_pG_p) \tag{63}$$

If Eq. (50) is divided by Eq. (49), there results

$$\frac{\mathbf{E}_p}{\mathbf{E}_g} = \frac{U_1 + jU_2}{V_1 + jV_2} = \frac{U \angle \theta_1}{V \angle \theta_2} \tag{64}$$

Let

$$\frac{U}{V} = \delta;\ \theta_1 - \theta_2 = \theta \tag{65}$$

Then

$$\frac{\mathbf{E}_p}{\mathbf{E}_g} = \delta \angle \theta \tag{66}$$

and

$$\frac{E_p}{E_g} = \delta \tag{67}$$

Equation (63) may now be written in the form

$$P = E_p^2\left(\frac{G_{cp}}{\delta} - G_p\right) \tag{68}$$

Now the power furnished to the oscillating circuit by the neutralizer has been shown to be equal to NI_1^2. If all of the power output of the triode goes into the oscillating circuit, we may write

$$P = NI_1^2 = E_p^2\left(\frac{G_{cp}}{\delta} - G_p\right) \tag{69}$$

But from Eq. (50) the plate voltage is proportional to the current in the oscillating circuit, *i.e.*,

$$\mathbf{E}_p = U \angle \theta_1 \mathbf{I}_1$$
$$E_p = UI_1 \tag{70}$$

Upon substituting Eq. (70) in Eq. (69), there results

$$N = U^2\left(\frac{G_{cp}}{\delta} - G_p\right) \tag{71}$$

As stated at the beginning of this section, one use for Eq. (71) is found in obtaining approximate expressions for N for certain resistance neutralization circuits. This merely requires that we find the value of U and of δ for the par-

ticular circuit for which N is desired. As an illustration consider again the circuit of Fig. 25. If the voltage induced in the plate circuit by the oscillating current I_1 is large compared to the plate voltage due to the plate space current, we may write

$$E_p = \omega M_p I_1$$

from which we see that for this circuit

$$U = \omega M_p \tag{72}$$

The alternating grid potential is given by the equation

$$E_g = \omega M_g I_1$$

The expression for δ then is

$$\delta = \frac{E_p}{E_g} = \frac{M_p}{M_g} \tag{73}$$

Upon substituting Eqs. (72) and (73) in Eq. (71), there results

$$N = \omega^2 M_p^2 \left(\frac{M_g}{M_p} G_{cp} - G_p \right)$$

$$= \omega^2 M_p (M_g G_{cp} - M_p G_p) \tag{74}$$

$$= \omega^2 h \tag{75}$$

In Eq. (74) M_p and M_g stand for the absolute values of the mutual inductances. The equation is valid only when M_p and M_g have opposite signs, because Eq. (71) is valid only when the plate and grid voltages are 180 degrees out of phase.

The exact expression obtained for N at the beginning of this chapter was

$$N = \frac{\omega^2 h D}{D^2 + \omega^2 L_p^2 G_p^2}$$

If, as is generally the case, $\frac{D}{D^2 + \omega^2 L_p^2 G_p^2}$ is nearly equal to unity, the approximate expression obtained so easily from Eq. (71) is very nearly correct.

As another example of the use of Eq. (71), consider the circuit shown by Fig. 27.

If the circuit is resonant to the generator frequency, I_1 will in general be large compared to I_p so that I_2 very nearly equals $-I_1$. If the choke coils around the condensers do not appreciably affect the plate and grid voltages, we have

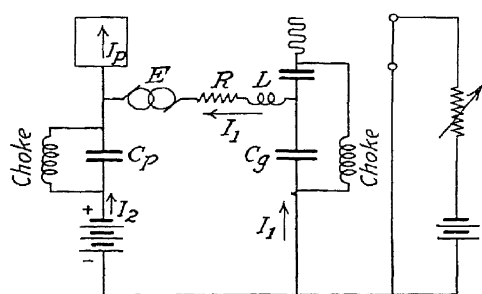



Fig 27.

$$\mathbf{E}_p = \frac{j\mathbf{I}_2}{\omega C_p} = -\frac{j\mathbf{I}_1}{\omega C_p} \quad (76)$$

$$\mathbf{E}_g = \frac{j\mathbf{I}_1}{\omega C_g}$$

The plate and grid voltages are thus seen to be 180 degrees out of phase.

$$\frac{E_p}{E_g} = \frac{C_g}{C_p} = \delta$$

Therefore if $\frac{C_g}{C_p} < \frac{G_{cp}}{G_p}$ the triode will lower the resistance of the circuit.

From Eq. (76)

$$U = \frac{1}{\omega C_p}$$

Therefore from Eq. (71), N for this circuit is given by

$$N = \frac{1}{\omega^2 C_p^2}\left(\frac{G_{cp}C_p}{C_g} - G_p\right)$$

$$= \frac{1}{\omega^2}\left(\frac{G_{cp}}{C_pC_g} - \frac{G_p}{C_p^2}\right) \quad (77)$$

17. Variation of N with the Amplitude of the Current in the Oscillating Circuit.

In the treatment of triode circuits which has been given so far, operation has been confined to plane portions of the characteristic surface of the triode. The equations which have been derived are correct only for this condition. If the portion of the characteristic surface over which operation takes place is but slightly curved, the equations given above apply very closely to actual conditions if the conductances are calculated at the operating point. In some cases, however, notably in the generation of sustained oscillations,

operation takes place over curved portions of the characteristic surface. A rigorous solution of the triode circuit problem under these conditions requires that the equation of the characteristic surface be written down either in finite form or in the form of an infinite series. Solutions based on an infinite series expression for the characteristic surface of a triode are very valuable for obtaining certain types of information. These solutions, however, are somewhat cumbersome to handle and do not give a simple picture of the operation of the triode when it is functioning as a generator of sustained oscillations. In the treatment given in this and a few of the following sections, we shall assume that Eq. (71) is valid even when operation takes place over portions of the characteristic surface which are curved, provided that the conductances G_{cp} and G_p are allowed to become functions of the amplitudes of the alternating plate and grid potentials. The justification for making this assumption is threefold: First, it leads to a simple picture of the action of a triode oscillator. Second, it enables one to calculate very closely the performance of a triode oscillator when operation does not take place over portions of characteristic curves which show pronounced saturation. Third, the treatment based upon the assumption that Eq. (71) is valid suggests the determination of some general experimental curves from which the performance of oscillators can be calculated with reasonable accuracy.

In Eq. (71), the circuit parameters U and δ are independent of the amplitude of the current in the oscillating circuit. N varies only because the conductances G_{cp} and G_p are taken as functions of the amplitudes of the alternating grid and plate potentials. This section will be devoted to a discussion of the variation of N with the amplitude of the current in the oscillating circuit or to the variation of N with the amplitude of the alternating plate potential. In order to have the curves which we are about to derive independent of any circuit, we shall plot $\frac{N}{U^2}$ instead of N as a function of the amplitude of the plate alternating potential.

If we divide both sides of Eq. (71) by $U,^2$ there results

$$\frac{N}{U^2} = \frac{G_{cp}}{\delta} - G_p \qquad (78)$$

In deciding upon a method of determining the conductances G_{cp} and G_p, the fact should be kept in mind that there is no exact way of determining them, because in the derivation of Eq. (78) these conductances were assumed to be constants. We shall therefore demand of the method which we adopt, first, that it be a simple one and, second, that it give fairly accurate results when operation takes place over portions of the characteristic curves which do not show a pronounced flattening out on either side of the operating point. The method adopted is to use the slope of the secant line connecting the extreme points of operation as illustrated below.

Let the characteristic curves for the triode under discussion be the curves of Fig. 6. Let the operating point be chosen where the steady plate potential has a value of +500 volts and the steady grid potential has a value of +40 volts. Let the plate potential remain constant at 500 volts and introduce into the grid circuit an alternating potential having a **peak** value of 5 volts. The grid potential then varies from 35 to 45 volts along the 500-volt plate potential curve. The controlled plate conductance will be taken as the slope of the secant line connecting these points; that is, the value assigned to G_{cp} when the alternating grid potential is 5 volts is

$$G_{cp} = 1{,}060 \times 10^{-6} \text{ mhos}$$

Now let the peak value of the alternating grid potential take on successively the values 10, 15, 20, 25 . . . 60 volts, and obtain values of the controlled plate conductance for each of these potentials. Upon plotting values of G_{cp} as ordinates and values of peak alternating grid potentials as abscissae, curve A of Fig. 28 is obtained.

Now using the data given by the curves of Fig. 6, plot a curve between plate current and plate potential when the grid potential has a constant value of +40 volts. If the grid potential is maintained constant at a value of +40

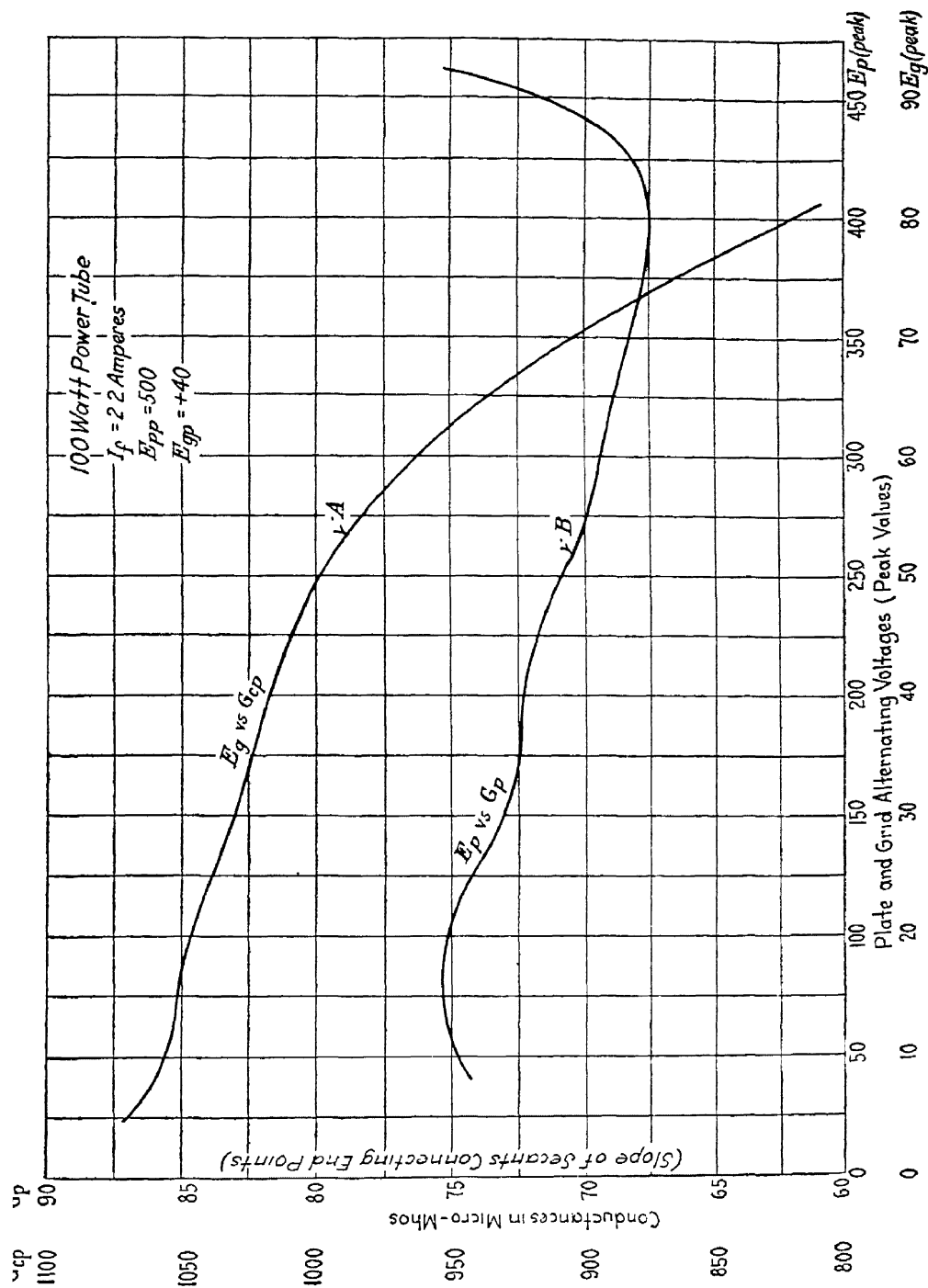



FIG. 28.—Variation of conductances with amplitude of plate and grid voltages.

volts, and an alternating potential having a peak value of 50 volts is impressed on the plate, operation takes place along the curve described above from 450 to 550 volts. The plate conductance for an alternating plate potential of 50

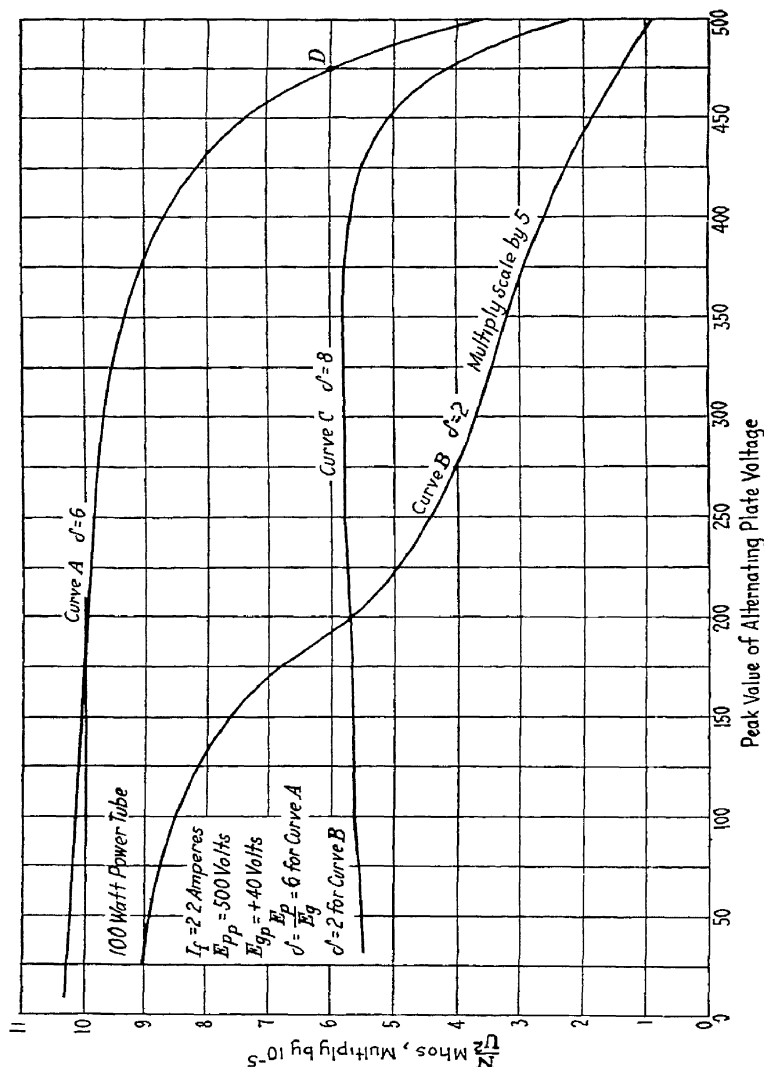



FIG 29.—Calculated $\frac{N}{U^2}$ curves

volts will be taken as the slope of the secant line connecting these points. Curve *B* of Fig. 28 was obtained by assigning a succession of values to the amplitude of the alternating plate potential and taking the slope of the secant line as described above in each case. This curve gives G_p as a

function of the peak value of the alternating plate potential when the grid potential is +40 volts.

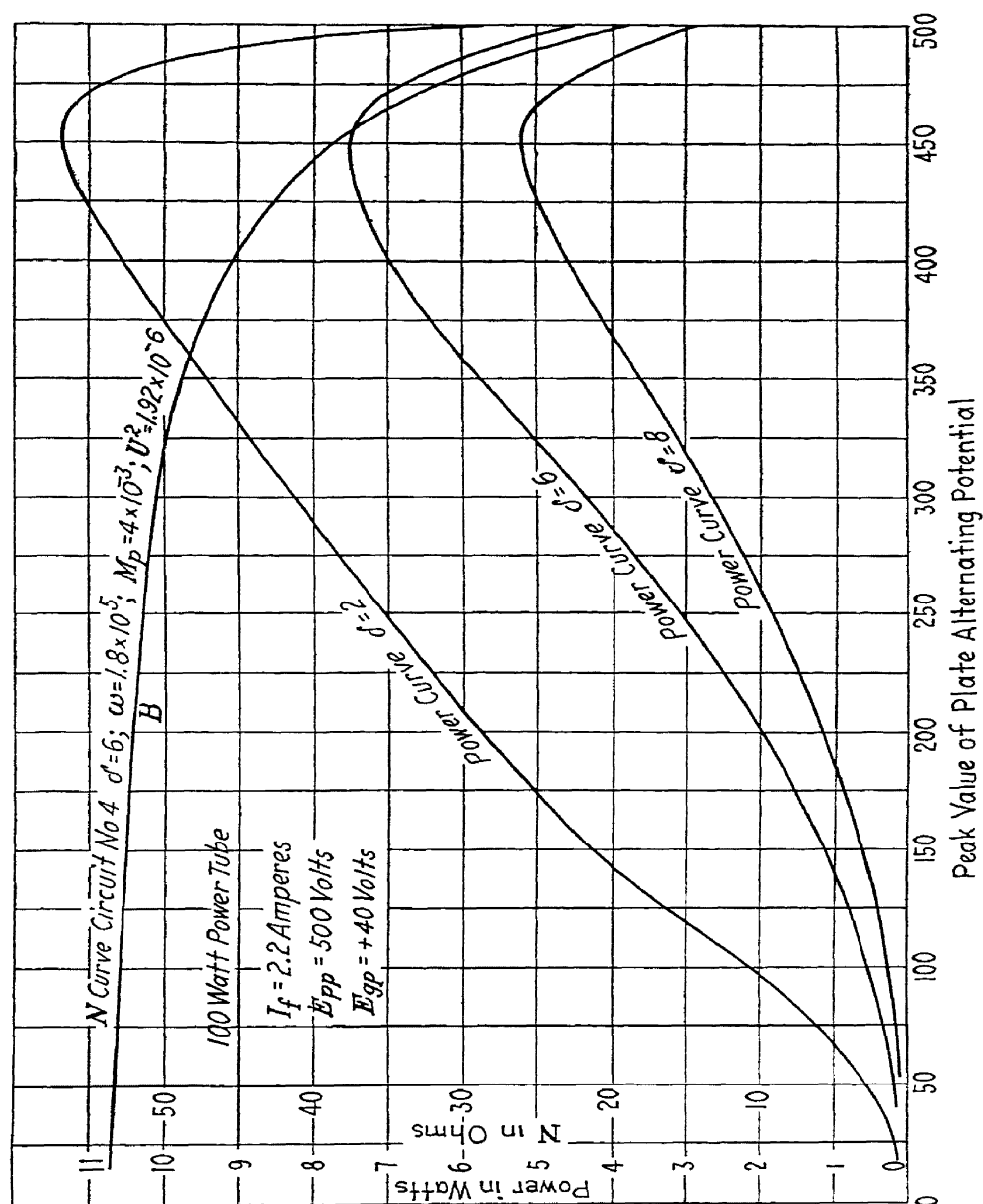



FIG 30.—Calculated power output curves and N curve.

Now let us assign the value 6 to δ (the best value for δ will be discussed in connection with the generation of sus-

tained oscillations), and use Eq. (78) and the curves of Fig. 28 to plot a relation between $\frac{N}{U^2}$ and E_p (peak value). A point on this curve is obtained as follows. Let E_p be taken as 200 volts. Then since $\delta = 6$, $E_g = {}^{200}/_6 = 33.3$ volts. From curve A of Fig. 28, $G_{cp} = 1{,}030 \times 10^{-6}$ when $E_g = 33.3$. $G_p = 72.5 \times 10^{-6}$ so that when the peak value of $E_p = 200$, then

$$\frac{N}{U^2} = \left(\frac{1{,}030}{6} - 72.5\right) \times 10^{-6} = 9.92 \times 10^{-5}.$$

The complete $\frac{N}{U^2}$ curve for $\delta = 6$ is given by curve A of Fig. 29. Curves B and C of the same figure give the $\frac{N}{U^2}$ curve for values of δ equal to 2 and 8, respectively.

The curves of Fig. 29 are independent of any circuit and are therefore general curves of the triode when operating at the point $E_{pp} = 500$; $E_{gp} = 40$.

To derive the N *vs.* E_p curves from the curves of Fig. 29 it is only necessary to know the value of U for the particular circuit under consideration and to multiply the values of $\frac{N}{U^2}$ by U^2 to obtain the values of N. Thus for the circuit of Fig. 25 we have shown that $U = \omega M_p$. The N *vs.* E_p (peak value) curve for this circuit when $\omega = 1.8 \times 10^5$ and $M_p = 4 \times 10^{-3}$ as derived from curve A of Fig. 29 is curve B of Fig. 30.

18. Power Output Curves.

When the $\frac{N}{U^2}$ curves of a tube are available, it is a very simple matter to obtain the power output curves, for

$$P = NI^2;\ I = \frac{E_p}{U}$$

$$P = \frac{N}{U^2}E_p^2 = \frac{NE_p^2 \text{ (peak value)}}{2U^2} \tag{79}$$

As an illustration of the use of Eq. (79) in plotting power output curves, let $\delta = 6$ and let the $\frac{N}{U^2}$ curve be curve A

of Fig. 29. When the peak value of the alternating plate voltage is 200, we find from the curve that $\frac{N}{U^2} = 9.92 \times 10^{-5}$. Substituting in Eq. (79), there results

$$P = (9.92)(10^{-5})\frac{(200)^2}{2} = 1.98 \text{ watts}$$

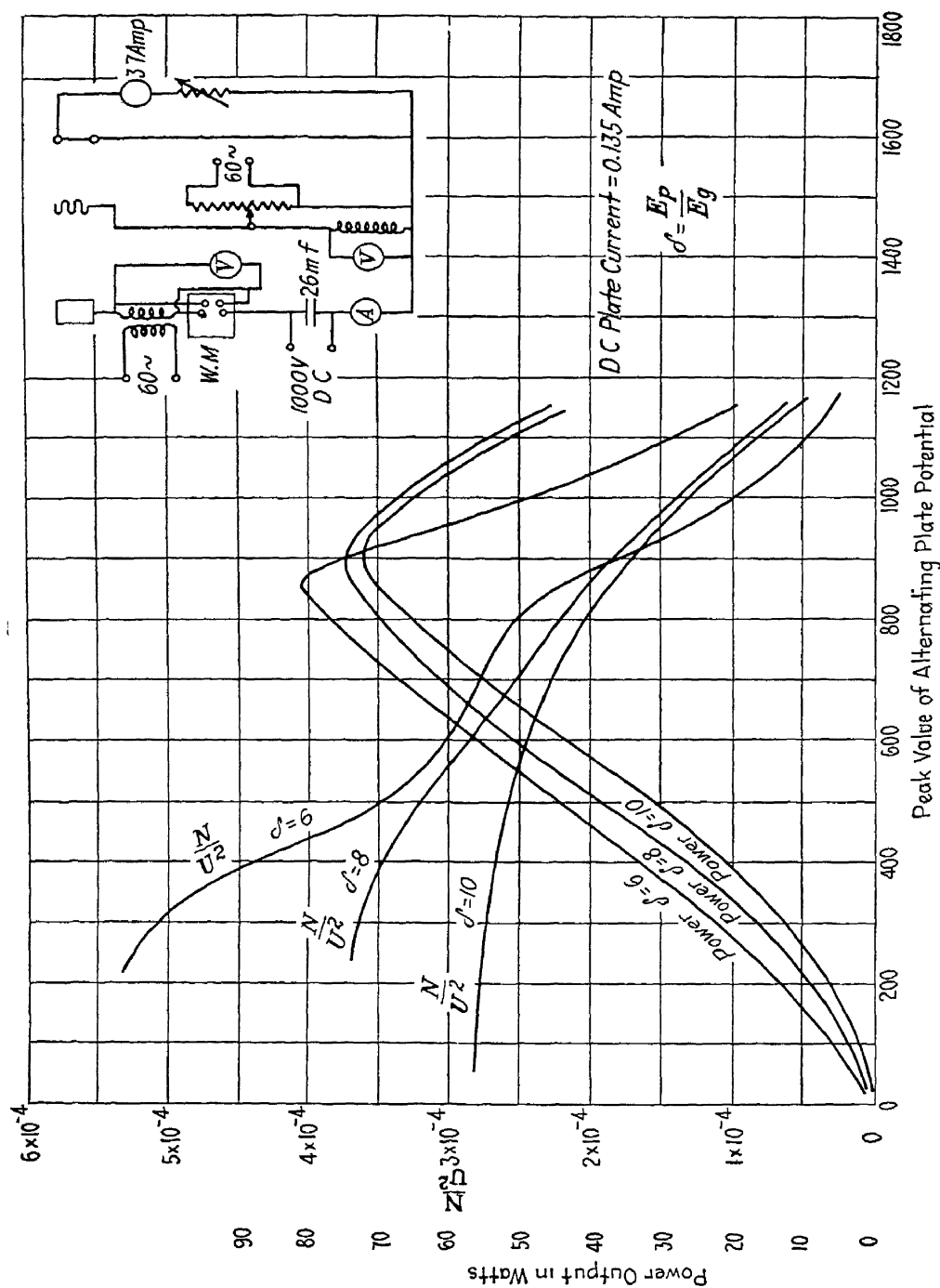
Calculated curves between power output and peak alternating plate voltage for three values of δ are given by Fig. 30. These curves give the total alternating power made available for use in the system external to the tube when the plate and grid alternating voltages are substantially 180 degrees out of phase.

19. Direct Experimental Determination of $\frac{N}{U^2}$ Curves.

In predicting the performance of a triode as a generator of sustained oscillations, it is necessary to have the $\frac{N}{U^2}$ curves for high plate and grid alternating voltages. It is impossible to run characteristic curves of large power tubes at high plate and grid voltages because of the heating of the plate. Furthermore, as pointed out in Sec. 17, the $\frac{N}{U^2}$ curves calculated from the characteristic curves are not very accurate for values of alternating plate and grid potentials which cause operation to take place over portions of the characteristic curves which show a pronounced saturation. It is therefore desirable to devise a method for experimentally determining these curves. The heating of the plate can be eliminated by using circuits which decrease the power expended on the plate. The power expended on the plate will be decreased if alternating potentials are impressed on the plate and grid which are 180 degrees out of phase and of such relative magnitude that

$$\frac{E_p}{E_g} = \delta < \frac{G_{cp}}{G_p}.$$

If the power output is then read on a wattmeter while E_p and E_g are varied in such a manner as to keep δ constant, we can plot directly power output curves similar to those

FIG 31 —Experimental performance curves for 250 watt pliotron

given by Fig. 30. The $\frac{N}{U^2}$ curves can be obtained from these power curves by making use of Eq. (79), for

$$\frac{N}{U^2} = \frac{2P}{E_p^2 \text{ (peak value)}} \tag{80}$$

The actual circuit used to obtain such a set of power curves is shown by the circuit of Fig. 31. This same figure sheet gives the power output curves and the $\frac{N}{U^2}$ curves for three values of δ. The tube used was a 250-watt power tube, and operation took place about a point where the continuous plate voltage was 1,000 and the continuous grid voltage was zero. Since a peak alternating plate potential of 1,200 volts was reached, the maximum total plate potential was 2,200 volts and the maximum grid potential was 200 when δ was equal to 6. The maximum point which it was possible to reach on the static characteristic curves for this same tube without melting the plate was a plate potential of 900 volts and a grid potential of 80 volts.

CHAPTER IV

THE TRIODE AS A GENERATOR OF SUSTAINED OSCILLATIONS

20. Conditions Leading to the Generation of Sustained Oscillations in Triode Circuits.

In the preceding chapter the general theory of resistance neutralization has been discussed in detail. It is the purpose of this chapter to apply the theory developed in Chap. III to a discussion of the operation of triode circuits when used to generate sustained oscillations.

Consider a simple series circuit consisting of an inductance, a capacitance, a resistance, a resistance neutralizer, and a switch. Let the switch be open and the condenser be charged so that the potential between its terminals is E_c. The differential equation for this circuit is Eq. (15) of Sec. 12 in which $e_B = 0$. The equation for the current which flows in the circuit immediately after the switch is closed is obtained by solving this equation subject to the conditions that the condenser voltage is E_c and the current is zero when time is zero. From analogy with the known solution of Eq. (16) of Sec. 12, we write

$$i_1 = \left[\frac{E_c}{\beta L} \sin \beta t\right] \epsilon^{-\frac{R-N}{2L}t} \tag{1}$$

$$\beta = \sqrt{\frac{1}{LC} - \frac{(R-N)^2}{4L^2}} \tag{2}$$

If R is greater than N, Eq. (1) is the ordinary equation of a damped sine wave as shown by Fig. 32. As N approaches R the damping becomes less and less, and when $N = R$, the damping is zero and the current has the form shown by Fig. 33. When N is greater than R, the exponential term becomes an amplifying factor and the oscillations continually grow in amplitude as shown by Fig. 34.

The conclusions arrived at above may also be obtained from energy considerations. The energy stored in the circuit at the moment of closing the switch is $\frac{1}{2}CE_c^2$. The power expended in the resistance is Ri^2. The power fed into the circuit by the neutralizer is Ni^2. The net power drawn from the original power storage $\frac{1}{2}CE_0^2$ is therefore equal to $(R - N)i^2$. If R is greater than N_1,

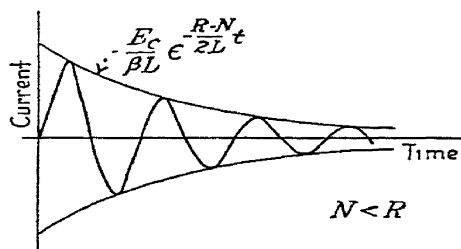



FIG 32.

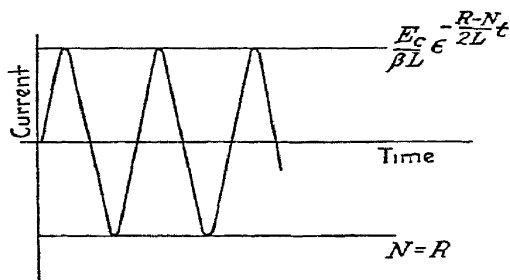



FIG 33

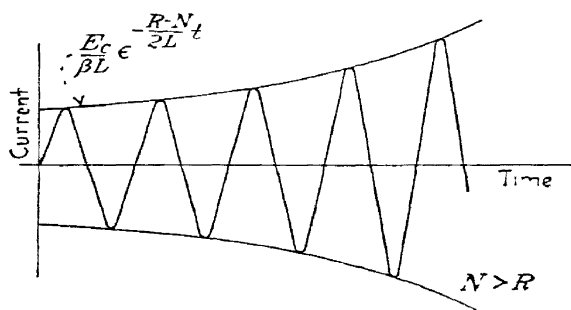



FIG 34.

the energy of the system is dissipated and the amplitude of the oscillations decreases with the time. If $R = N$, the energy transfer from the capacitance to the inductance and back again remains constant and the amplitude of the oscillations remains constant. If N is larger than R, more power is fed into the circuit than is dissipated in the resistance, and the energy of the electrostatic and the magnetic fields continually increases. Therefore the oscillations must grow in value, since the energy of the magnetic field is $\frac{1}{2}Li^2$.

21. Amplitude of the Oscillations.

The case of importance in the generation of sustained oscillations is the case for which N is initially greater than R. The oscillations start to build up as shown by Fig. 34. As the current in the oscillating circuit grows in amplitude, so also do the plate and grid voltages if the neutralizer is a triode. This requires that operation take place over an increasing area of the characteristic surface of the triode. When the amplitude of the oscillating current reaches a certain value, N commences to decrease with an increase in the amplitude of the oscillating current. When the oscillations grow to such a value that N and R are equal, the power supplied to the circuit is just equal to the power dissipated, and the oscillations remain fixed at this amplitude.

We can determine just what this amplitude will be from the $\frac{N}{U^2}$ curves developed in Chap. III. Let R be the total resistance of the system referred to the oscillating circuit, and let the system be such that the plate and grid alternating voltage are substantially 180 degrees out of phase. Further suppose that the value of U and of δ are known for the system and that the $\frac{N}{U^2}$ curve is available for the value of δ which obtains in the system. The alternating plate voltage will then build up until $\frac{R}{U^2} = \frac{N}{U^2}$ This value can be read from the $\frac{N}{U^2}$ curve. Then since E_p (peak value) = U (I_1 peak value), the r.m.s. value of the oscillating current will be given by the relation

$$I_1 \text{ (r.m.s.)} = \frac{E_p \text{ (peak)}}{\sqrt{2}U} \tag{3}$$

As an example of the method of determining the amplitude of the oscillations, let the triode be the 100-watt tube whose characteristics are given by the curves of Fig. 6. Let the steady plate voltage be 500 and the steady grid

voltage be +40. Then its $\frac{N}{U^2}$ curves are given by Fig. 29. Let the oscillating system be the circuit of Fig. 25 with the alternator removed. Let the constants of the circuit be such that its natural angular velocity is 1.8×10^5 radians per second. Let $M_p = 4 \times 10^{-3}$ and $M_g = 6.66 \times 10^{-4}$ henrys. Then $\delta = \frac{M_p}{M_g} = 6$, $U = \omega M_p = 7.2 \times 10^2$, $U^2 = 5.19 \times 10^5$. If the resistance of the system referred to the oscillating circuit is 51.9 ohms, then $\frac{R}{U^2} = 10^{-4}$ mhos. From the curve of Fig. 29 for which $\delta = 6$, we see that when $\frac{N}{U^2} = 10^{-4}$, the peak value of the alternating plate potential is 200 volts. Upon applying Eq. (3) we find that the r.m.s. value of the current in the oscillating circuit 1 will be 200 divided by $\sqrt{2}(7.2) \times 10^2 = 0.196$ amperes.

22. Stability of Operation.

Let the $\frac{N}{U^2}$ curve for the case under discussion be curve A of Fig. 29. It is evident that if the effective resistance R of the oscillating system referred to the oscillating circuit 1 is such that $\frac{R}{U^2}$ is greater than 1.05×10^{-4}, any disturbance in the system will be damped out, since this value of $\frac{R}{U^2}$ is greater than any possible value of $\frac{N}{U^2}$, and $R - N$ would always be positive. For any value of $\frac{R}{U^2}$ greater than 9.5×10^{-5}, the oscillations would be unstable, as any slight change in the value of the tube or circuit constants would cause the oscillations to vary over a wide range in order to keep $R = N$. When $\frac{R}{U^2}$ is less than 9.5×10^{-5}, operation will be stable. For instance if $\frac{R}{U^2} = 6 \times 10^{-5}$ mhos, the oscillations would build up until the peak value of the plate alternating voltage reached 475 volts. The resistance or

circuit constants could vary so that $\frac{R}{U^2}$ ranged from 2×10^{-5} to 8×10^{-5} without causing an inordinate change in the amplitude of the oscillations. The operation of the triode on its $\frac{N}{U^2}$ curve is in some respects similar to the operation of a shunt generator at points on its magnetization curve. If operation is at a point around the knee of the curve, any variation from the operating point sets up reactions which oppose the change, and the operation is stable.

In the case under discussion, if operation is taking place about the point D and the oscillations grow larger, $\frac{N}{U^2}$ becomes less, $R - N$ becomes positive, and the amplitude of the oscillation is diminished. If on the other hand the amplitude of the oscillation decreases in value, $\frac{N}{U^2}$ increases, $R - N$ becomes negative, and the amplitude is restored to the value for which $R - N = 0$. A consideration of the above discussion will show that stable operation takes place over those portions of the $\frac{N}{U^2}$ curve which have a large negative slope.

23. Power Output.

When the amplitude of the oscillations has been obtained, the power output of the triode generator is readily obtained because

$$P = NI_1^2 = RI_1^2 \tag{4}$$

where R is the equivalent resistance of the system referred to the oscillating circuit.

The power output curves for a triode functioning as a generator of sustained oscillations are the same as those developed in Chap. III, Sec. 18. Such a set of curves is given by Fig. 30. These curves show that the power output depends upon the value of the plate alternating potential and upon the value of δ. As we have seen before in the discussion of the amplitude of the oscillations, the plate alternating potential depends upon the value of $\frac{R}{U^2}$.

Therefore the power output for any given operating point depends upon the values of R, U, and δ.

If the value of δ is fixed and if we have a power output curve and a $\frac{N}{U^2}$ curve for this value of δ, it is an easy matter to find what value to assign to $\frac{R}{U^2}$ for maximum power output. The value of the plate alternating potential for maximum power output can be read off from the power curves. Then the value of $\frac{N}{U^2} = \frac{R}{U^2}$ which will cause the plate potential to build up to this value can be obtained from the $\frac{N}{U^2}$ curve.

As an example let triode be the 100 watt power tube and let operation take place at such a point that its $\frac{N}{U^2}$ curves are given by Fig. 29 and its power output curves by Fig. 30. Let $\delta = 6$. Then from the power output curve, the peak value of the alternating plate potential for maximum power output is found to be 445 volts. From the $\frac{N}{U^2}$ for $\delta = 6$, we find that the oscillations will build the plate potential up to 445 volts if $\frac{R}{U^2} = 7.6 \times 10^{-5}$. If the oscillating system is the one shown by Fig. 25 with the generator absent and if the constants are those given in Sec. 21 of this chapter, then $U^2 = 5.19 \times 10^5$, and the equivalent resistance of the oscillating circuit for maximum power output must be $\left(\frac{R}{U^2}\right)U^2 = (7.6 \times 10^{-5})(5.19 \times 10^5) = 39.4$ ohms.

The results obtained using the $\frac{N}{U^2}$ curves obtained by the wattmeter method give more accurate results than the $\frac{N}{U^2}$ curves derived from the characteristic curves using values of the conductances equal to the slopes of the secant lines. This is particularly true for the lower values of δ. In this case a review of the methods of calculating the $\frac{N}{U^2}$ curves

will show that at the higher plate and grid alternating potentials, the value assigned to G_{cp} is generally too low, and the value assigned to G_p is too high. This reduces the calculated power output to a value which is lower than the one obtained in oscillating systems.

24. Conditions for Maximum Power Output and Efficiency.

The general equation for the power output of a triode has been shown to be

$$P = \frac{1}{2} E_p I_p \cos \theta \tag{5}$$

where E_p is the peak value of the alternating plate potential I_p is the peak value of the alternating plate space current, and θ is the angle between the plate current and the plate voltage. For maximum power output $\cos \theta$ should equal -1. This requires that the plate and grid voltages be 180 degrees out of phase and that $E_g G_{cp}$ is larger than $E_p G_p$.

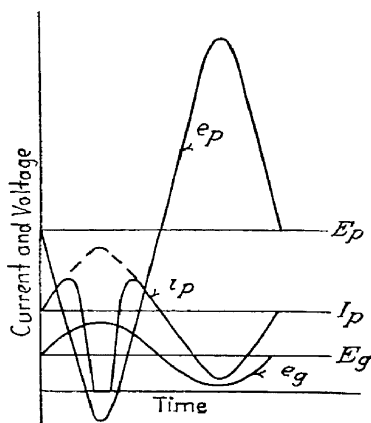



FIG 35.

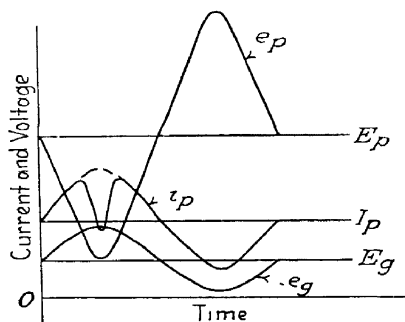



FIG. 36

This follows from the discussion of Sec. 10, Chap. II. When these conditions are fulfilled, Eq. (5) shows that maximum power output obtains when the plate voltage and plate current have the largest feasible variation. Now the peak value of the plate alternating voltage should not exceed the continuous voltage applied to the plate, nor should it be of such a value as to make the total plate poten-

tial fall much below the total grid potential during the negative half cycle of the plate alternating voltage; for when these conditions obtain, but few electrons can reach the plate and the conditions are as shown by Figs. 35 and 36. In the positive half cycle of the plate current wave the current decreases to low values or even to zero just where it

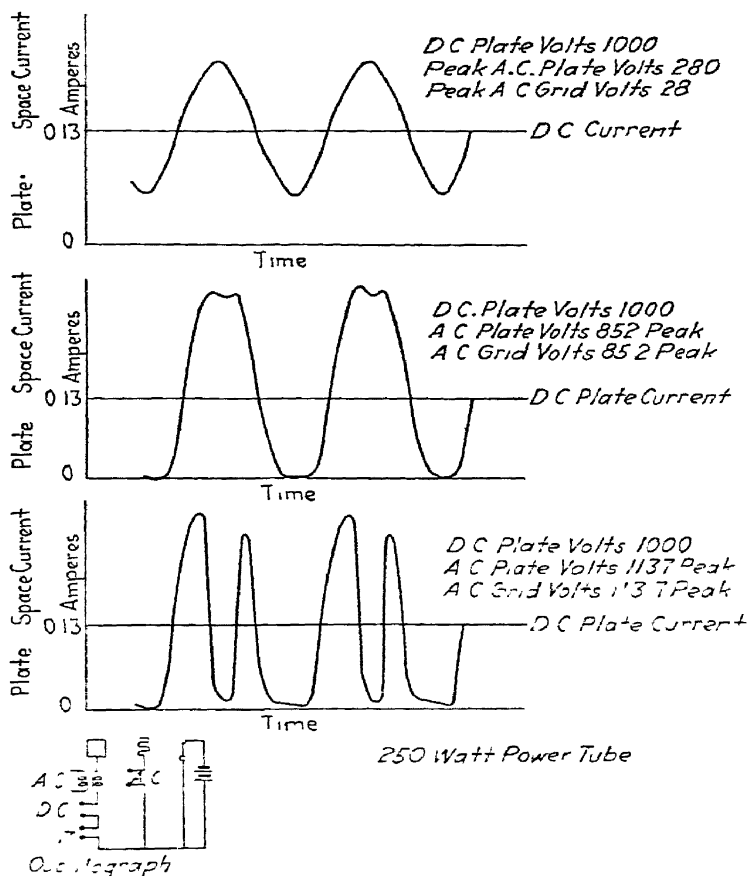



FIG 37 —Oscillograms showing effect of increased plate voltage on plate current

should have large values, and if the conditions are bad enough, the total plate current may fall to zero. This condition might lead to high efficiency by lowering the input from the direct-current battery or generator supplying the plate space power, but it would so decrease the alternating power output as to render it unworthy of much considera-

tion. Figures 35 and 36 have been sketched roughly from a consideration of the number of electrons which could reach the plate. The actual plate space current curves of Fig. 37 bring out this condition very strikingly. These curves were sketched on the oscillograph tracing table. The circuit used is shown on the curve sheet. The plate and grid voltages are 180 degrees out of phase. In the first curve the minimum value of the plate potential is 1,000 − 280 = 720 volts, and at the same instant of time the grid voltage is 25 + 20 = 45, so the total plate voltage is always far above the total grid voltage, and the alternating plate space current is substantially a sine wave. In the second tracing the total plate potential falls to 1,000 − 852 = 148 volts and the grid potential at the same instant of time is 85.2 + 20 = 105.2. The plate space current wave is just commencing to have a dip in the positive peak. In the third tracing, the total plate potential falls to 1,000 − 1,137 = −137 volts, and as predicted the plate space current falls nearly to zero just where its maximum value should come for high power output. Because of these conditions the peak value of the alternating plate potential must be limited to 0.9 or less of the continuous plate potential.

The negative peak of the alternating plate current cannot exceed the continuous plate space current. Operating points are generally so chosen that saturation limits the positive current peak to the same value. Therefore in most cases the peak value of the alternating plate space current is limited to the value of the continuous plate current. If it is desired to keep the plate space current approximately sinusoidal, the limiting value of the power output would be

$$P \text{ (limiting value, sine-wave operation)} = \tfrac{1}{2}E_{pp}I_{pp} \quad (6)$$

The power input to the plate space is

$$P \text{ (input to plate space)} = E_{pp}I_{pp} \quad (7)$$

The limiting value of the plate space efficiency with sine wave operation is

$$\frac{\tfrac{1}{2}E_{pp}I_{pp}}{E_{pp}I_{pp}}100 = 50 \text{ per cent} \quad (8)$$

This is the limiting value of the efficiency and seldom can be reached in practice with sine-wave operation.

It was stated above that the peak value of the plate space current was in general limited to the value of the continuous plate space current. While this is true, its r.m.s. value can also be made to approach this same value by making the wave rectangular in form. If the plate voltage is a sine wave as is generally the case, then only the first harmonic of the current wave contributes to the power output. If I_{pp} is the continuous plate current and if the alternating plate current is rectangular, the peak value of the first harmonic of the current wave is $\frac{4I_{pp}}{\pi}$. If the peak value of the plate alternating voltage is taken equal to the continuous plate voltage, the maximum power output is $\frac{2E_{pp}I_{pp}}{\pi}$. The input to the plate space is as usual $E_{pp}I_{pp}$. The limiting value of the plate space efficiency is

$$\frac{2E_{pp}I_{pp}}{\pi E_{pp}I_{pp}}100 = \frac{200}{\pi} = 63.8 \text{ per cent} \qquad (9)$$

This efficiency does not take into account the losses in the grid-filament space or the power required to heat the filament.

We have seen that the power output will be a maximum when E_p and I_p have the largest possible values. Now as far as the alternating grid voltage is concerned, I_p will have the largest possible value when E_g is as large as feasible. The value of E_g is limited by the following considerations:

1. The total grid potential may not exceed the voltage which causes flashover between the grid and filament.
2. The grid alternating voltage may become so high that, during the negative half cycle of the plate voltage, the total plate potential falls below the total grid potential.
3. The losses due to the grid conductance may become excessive.
4. The distortion due to a rectangular plate current wave may in some cases be objectional.

Condition 1 is self-evident and needs no further amplification.

When the triode is operating as a generator, the grid has its maximum total potential at the same instant of time when the total plate potential is a minimum. If the plate potential falls much below the grid potential, the plate current wave has a dip in its positive half cycle as shown by Fig. 36. This leads to a decrease in power output if it is very marked. The condition shown by Fig. 36 can be overcome by lowering either the plate or the grid alternating potentials or both. If the grid alternating voltage is already so high that the plate current wave is flat topped, the power output will be lowered less by decreasing the grid alternating voltage rather than the plate alternating voltage, because when this condition obtains, the first harmonic power component of the plate current decreases slowly with a decrease in grid alternating voltage.

Conditions 3 and 4 will not be discussed further in this section.

Let us now assume that we have assigned the largest feasible value to the grid alternating voltage. What value should we assign to the plate alternating voltage, that is, to δ, in order to obtain maximum power output? When the grid and plate voltages are displaced by 180 degrees, we have shown that the power output of a triode is given by

$$P = |E_p||E_g|G_{cp} - |E_p|^2 G_p$$

$$\frac{\partial P}{\partial E_p} = |E_g|G_{cp} - 2|E_p|G_p$$

The output will be a maximum when

$$\frac{\partial P}{\partial E_p} = 0$$

This occurs when

$$|E_p| = \frac{G_{cp}}{2G_p}|E_g| \tag{10}$$

or when

$$\delta = \frac{G_{cp}}{2G_p} \tag{11}$$

Now the peak value of the plate alternating voltage should never exceed the continuous plate voltage nor should the value of the plate alternating voltage be such as to cause the total plate potential to fall much below the total grid potential during the negative half cycle of the plate voltage. Subject to these two limitations the plate voltage should be as near the value given by Eq. (10) as possible. The value of δ then should be such that when the grid voltage has the largest feasible variation, the plate voltage has a value as near to that given by Eq. (10) as the limitations stated will permit. The best values of R and the circuit constants are such that $\frac{R}{U^2}$ has a value that will give to E_g the largest feasible variation. This value of $\frac{R}{U^2}$ is

$$\frac{R}{U^2} = \frac{N}{U^2} = \frac{G_{cp}}{\delta} - G_p \tag{12}$$

where G_{cp} is taken for the largest feasible value of E_g. G_p is taken for the corresponding value of $E_p = \delta E_g$ and δ is fixed by the considerations already discussed.

The material given in this section serves merely as a general guide in the designing of oscillating circuits and aids in obtaining an understanding of what is taking place in the circuits. For more accurate work the $\frac{N}{U^2}$ curves and power curves should be used as shown in Sec. 23. For large power tubes the experimental curves discussed in Sec. 19 of Chap. III lead to the most accurate design of oscillating systems when the theory of Sec. 23 of this chapter is applied.

25. Frequency of the Oscillations.

Let us consider that we have an electrical system in the ith branch of which there is situated a resistance neutralizer. Let the impedance of the system referred to this branch be $A_i + jB_i$. If a frequency can be found for which this impedance is zero, the system will generate sustained oscillations at this frequency. Now the term A_i will be of the

form $a_i - N_i$. The total losses in the system will be $a_i I_i^2$. The power furnished the system by the neutralizer will be $N_i I_i^2$. Therefore when $A_i = 0$, $a_i I_i^2 = N_i I_i^2$, and all of the losses in the system are supplied by the neutralizer as they should be, B_i must also be zero, for the total impedance must be zero, since nowhere in the system is there an applied voltage having the same frequency as the oscillation. For the simpler systems, the frequency of the oscillations will be very near to the natural frequency of the main oscillating circuit.

As an example consider again the circuit of Fig. 25. From Eq. (10), Sec. 11, we see that the impedance of the system referred to the oscillating circuit 1 is

$$A_i + jB_i = R_1 - \frac{\omega^2 hD}{D^2 + \omega^2 L_p^2 G_p^2} + j\left\{\omega L_1 - \frac{1}{\omega C_1} + \frac{\omega^3 L_p G_p h}{D^2 + \omega^2 L_p^2 G_p^2}\right\} \quad (13)$$

This system will oscillate so that both the real and the j terms disappear. The real term will disappear when $R_1 = N$ in accordance with the theory discussed in the preceding sections of this chapter. The actual frequency of oscillation will be such as to make the j term also disappear. The third quantity, however, in the j term is, as has been shown, generally small compared to the other two terms; so the frequency of oscillation is nearly the same as the frequency which will reduce $\omega L_1 - \frac{1}{\omega C_1}$ to zero, namely,

$$f = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

26. Operation of Oscillator Tubes in Parallel.

The system to be considered in this section is shown schematically by Fig. 38. The plates of the two tubes are connected directly together and so also are the two grids. The box A represents any combination of circuit elements arranged into an oscillating system. Since the grids are tied directly together, the potential of both grids is the

same. As before, the grid alternating potential will be represented by the symbol E_g. The plate alternating potential is represented by the symbol E_p. The plate and grid potentials are assumed to be 180 degrees out of phase.

Fig 38.

Now the total power supplied to the box A is equal to the power output of tube 1 plus the power output of tube 2, or

$$P = E_p I_{p1} + E_p I_{p2} \tag{14}$$

By the aid of Eq. (68) of Sec. 16, this power may be written in the form

$$P = E_p^2\left[\left(\frac{G_{cp1}}{\delta} - G_{p1}\right) + \left(\frac{G_{cp2}}{\delta} - G_{p2}\right)\right] \tag{15}$$

If essentially all of this power goes into the oscillating system, we may write as in Sec. 16

$$P = NI_1^2 = U^2 I_1^2\left[\left(\frac{N}{U^2}\right)_1 + \left(\frac{N}{U^2}\right)_2\right] \tag{16}$$

$$\frac{N}{U^2} = \left(\frac{N}{U^2}\right)_1 + \left(\frac{N}{U^2}\right)_2 \tag{17}$$

That is, the value of $\frac{N}{U^2}$ for the two tubes in parallel corresponding to any alternating plate voltage E_p is equal to the value of $\frac{N}{U^2}$ for tube 1 corresponding to the plate voltage

E_p plus the value of $\frac{N}{U^2}$ for tube 2 at this same plate voltage. In other words, if we have an $\frac{N}{U^2}$ *vs.* E_p curve for tube 1 and a corresponding curve for tube 2, the $\frac{N}{U^2}$ *vs.* E_p curve for the two tubes in parallel is obtained by adding the ordinates of the curves for the two tubes. When the $\frac{N}{U^2}$ curve for the two tubes in parallel is available, the theory of Secs. 17 to 25 applies for obtaining the amplitude of the oscillations, the conditions for maximum power output, and the frequency of the oscillations for the two tubes operating in parallel.

In changing from an oscillating system employing one tube to one which employs two tubes, there are some interesting things to be observed. Let us suppose that the single-tube oscillator has been so proportioned as to give maximum power output, and let us further suppose that $\frac{R}{U^2}$ for this system is equal to D. Now let a tube identical to the first one be connected in parallel in the circuit. The $\frac{R}{U^2}$ for maximum power will now be $2D$, and therefore in order to obtain maximum power from the two tubes in parallel, either R or U or both, will have to be changed in order to make $\frac{R}{U^2}$ equal to $2D$.

As an illustration, let the $\frac{N}{U^2}$ curves and the power output curves for each of the tubes under consideration be given by Fig. 31. Let the oscillating system be the circuit of Fig. 25 with the alternator removed. For this circuit $U = \omega M_p$ and $\delta = \frac{M_p}{M_g}$. If $\delta = 6$, we find from the power curves of Fig. 31 that the maximum power output of 81 watts occurs when the peak alternating plate potential is equal to 850 volts. From the $\frac{N}{U^2}$ curves on this same figure sheet,

we find that the value of $\frac{N}{U^2}$ must be equal to 2.3×10^{-4} in order to have an alternating plate voltage equal to 850. Then since $\frac{R}{U^2}$ must equal $\frac{N}{U^2}$, we write

$$\frac{R}{\omega^2 M_p^2} \text{ must equal } 2.3 \times 10^{-4}$$

in order to obtain maximum power from the single-tube oscillator. Now if a tube identical with the triode already in the circuit is connected in parallel with the first tube, the curves for the two triodes in parallel are obtained by doubling all of the ordinates of the curves of Fig. 31. If the second tube is connected into the circuit without making any other changes, we have

$$\frac{N}{U^2} = \frac{R}{\omega^2 M_p^2} \text{ still equals } 2.3 \times 10^{-4}$$

$\frac{N}{U^2}$ for the two tubes in parallel will have this value when the peak alternating plate potential has a value of 975 volts. The power output of the two tubes in parallel as obtained from the curves of Fig. 31 will be $2 \times 55 = 110$ watts. If we double the resistance in the circuit, however, or if we decrease M_p and M_g by the factor $\frac{1}{\sqrt{2}}$, then

$$\frac{R}{\omega^2 M_p^2} = \frac{N}{U^2} = 4.6 \times 10^{-4}$$

and the alternating plate potential (peak value) will build up to 850 volts and the power delivered to the system by the two tubes will be $2 \times 81 = 162$ watts.

Problems

9. In the circuit shown by Fig. 26, the device B is an alternator delivering the voltage $e_B = \sqrt{2} \cos \omega t$ The constants of the circuit are as follows:

$$L = 2 \times 10^{-3} \text{ henrys}$$
$$C = 2 \times 10^{-9} \text{ farads}$$
$$R = 10 \text{ ohms}$$

a. Calculate the natural angular velocity ω_r and the natural frequency f_r of the circuit.

b. Calculate the power delivered to the resistance with the neutralizer absent when

(1) $\omega = \omega_r$

(2) $\omega = (1.05)\omega_r$

(3) What is the ratio of the power P_r delivered to the resistance at resonance to the power P_d delivered to the resistance when $\omega = (1.05)\omega_r$?

c. The neutralizer A is now introduced into the circuit. The voltage introduced into the circuit by the neutralizer is $e_A = 9.9i_1$. Calculate the following:

(1) The power supplied by the generator when $\omega = \omega_r$

(2) The power supplied by the neutralizer when $\omega = \omega_r$

(3) The total power supplied to R when $\omega = \omega_r$

(4) The value of the reduction factor γ

(5) The regenerative amplification

(6) The total power supplied to R when $\omega = (1.05)\omega_r$

(7) The ratio of P_r as given by item (3) to P_d as given by item (6)

(8) The ratio of the power supplied by the neutralizer to the power supplied by the generator when $\omega = \omega_r$

(9) The power supplied by the generator when $\omega = (1.05)\omega_r$

(10) The ratio of the power supplied by the generator with the neutralizer out of the circuit to the power supplied by the generator with the neutralizer in the circuit when

(*a*) $\omega = \omega_r$

(*b*) $\omega = (1.05)\omega_r$

Give a brief discussion of the results obtained in this problem.

10. Work out and discuss the equations for the circuit shown by Fig. 39. In the discussion state the conditions under which resistance neutralization is obtained Show by means of a vector diagram that these conditions correspond to the conditions necessary for the triode to feed power into the circuit. (Use I_1 as a reference vector and for the purpose of obtaining the plate voltage on the diagram consider $I_2 = -I_1$.)

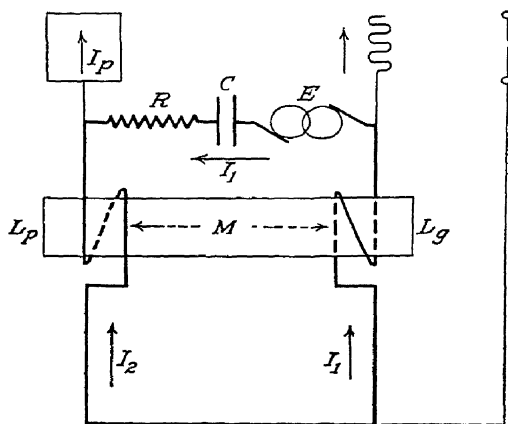



FIG 39

By what amount does heating the filament change the effective resistance of the circuit? By what amount does heating the filament change the effective reactance of the generator circuit?

Show by means of a vector diagram that if I_1 is large compared to I_p and if the ratio of reactance to resistance in each coil is large, then the resistances of both coils may be thought of as part of the resistance in the alternator branch without appreciable error.

11. From the characteristic curves for the 250-watt tube given by Fig. 40, derive the following curves:

(a) Peak alternating plate voltage as abscissa and G_p as ordinates
(b) Peak alternating grid voltage as abscissa and G_{cp} as ordinates

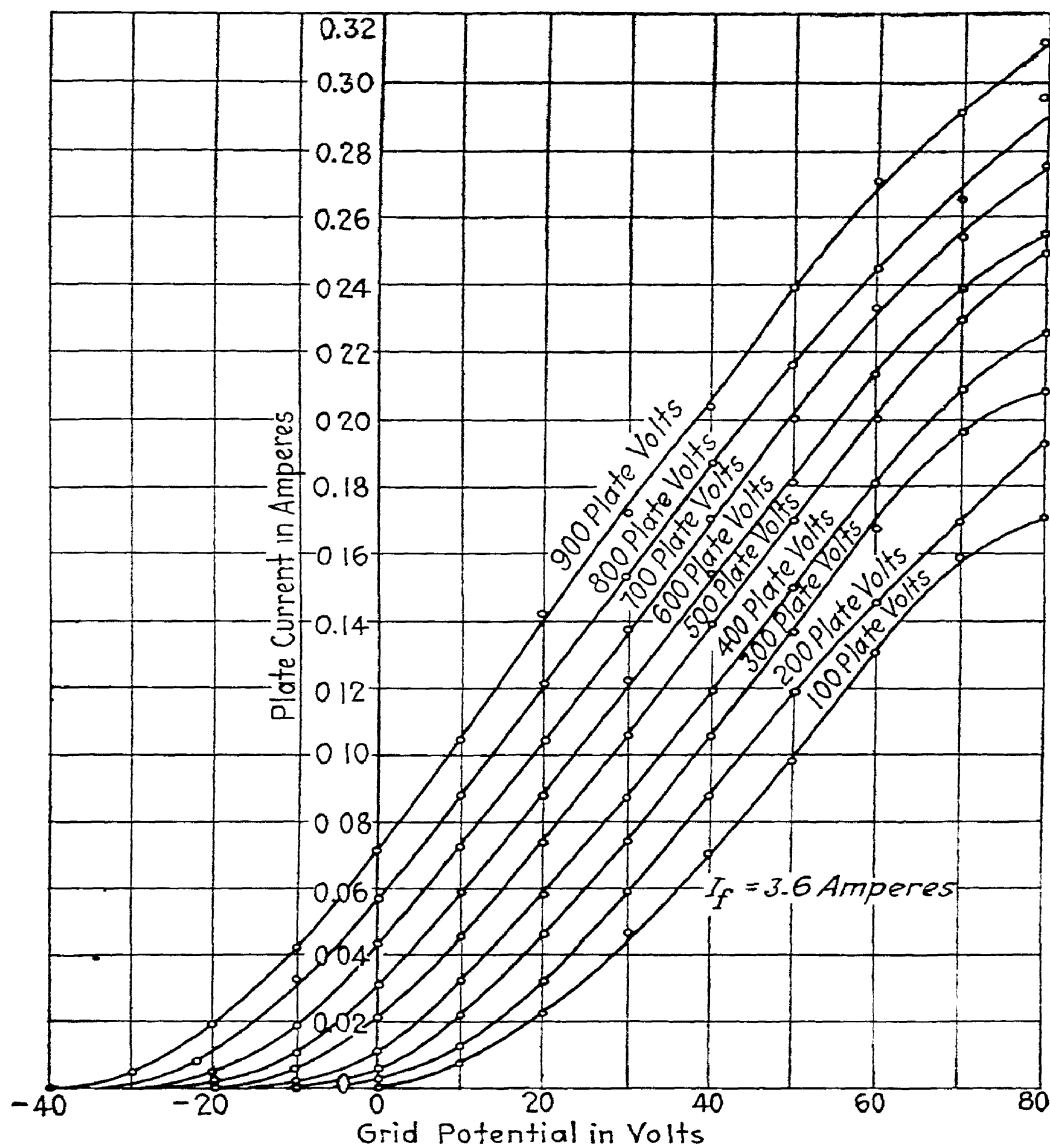



FIG. 40.—Characteristic curves for 250-watt power tube

(c) Peak alternating plate voltage as abscissa and $\frac{N}{U^2}$ as ordinates for values of δ = 10, 8, and 6.

The continuous plate voltage equals 700 and the continuous grid voltage equals plus 20.

12. If in any circuit the 250-watt tube is generating sustained oscillations when operating at the same point as given in problem 11, plot values of alternating plate voltage as abscissa and values of power output as ordinates for values of δ equal to 10, 8, and 6.

13. The circuit used for generating sustained oscillations is the circuit of Fig. 39 with the alternator removed. The circuit constants are:

$L_g = 0.002$ henry $L_p = 0.016$ henry
$M = 0$ $C = 10^{-8}$ farads

What is the value of δ? of U? What is the frequency of the oscillating current? What should be the equivalent resistance of the load and coils in order to obtain maximum power output? What is the value of this maximum power? What will be the amplitude of the current in the oscillating circuit, of the grid alternating voltage, of the plate alternating voltage when the resistance for maximum power is in the circuit? Will the oscillations be stable? The tube data are given in problem 11.

14. Work problem 13 using the $\frac{N}{U^2}$ curves and the power output curves given in Fig. 31. What changes should be made in the circuit constants if two tubes are operated in parallel when the curves of each tube are the curves given in Fig. 31?

CHAPTER V

BEHAVIOR OF RADIO RECEIVING SYSTEMS TO SIGNALS AND TO INTERFERENCE

27. Steady-state Properties of a Simple Series Antenna Circuit Associated with a Pure Resistance Neutralizer.

By the steady-state properties of a system of circuits we mean the behavior of the system to applied alternating voltages of constant amplitude and frequency after these voltages have been applied for so long a time that the currents everywhere in the system have reached their ultimate (steady-state) values. In this section we have under consideration the simple series antenna circuit shown by Fig. 41. This circuit consists of an antenna which has an effective capacitance to earth C in series with an inductance L, a wasteful resistance R_w, a detector whose resistance is R_d, and a resistance neutralizer which reduces the effective resistance of the circuit by the amount N. We fix our attention on two of the transmitting stations which are inducing voltages in the receiving antenna. We wish to receive energy from one of these stations and to receive just as little energy as possible from the other. The station from which we wish to receive energy will be called the **correspondent** station and we will designate its frequency by f_c. The other station will be called the **interferent** station. We will denote its frequency by f_i. We will assume that both of these stations are continuous wave stations and that their keys have been held down and will be held down for an indefinitely long time. Let each station induce the r.m.s. voltage E in the receiving antenna. If Z_c represents the impedance of the receiving antenna circuit to the correspondent frequency, the current which

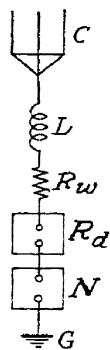



FIG 41

flows in the detector due to the correspondent station is

$$\mathbf{I}_c = \frac{\mathbf{E}}{\mathbf{Z}_c} \tag{1}$$

The power delivered to the detector due to the correspondent waves is

$$P_c = I_c^2 R_d = \frac{E^2 R_d}{Z_c^2} \tag{2}$$

The current in the circuit due to the interferent waves is

$$\mathbf{I}_i = \frac{\mathbf{E}}{\mathbf{Z}_i} \tag{3}$$

The power delivered to the detector due to the interferent waves is

$$P_i = I_i^2 R_d = \frac{E^2 R_d}{Z_i^2} \tag{4}$$

Let the **steady-state selective coefficient** of a radio receiving system against waves of any interferent frequency f_i be defined as the ratio of the steady-state power delivered to the detector due to waves of the correspondent frequency f_c to the steady-state power delivered to the detector due to waves of the interferent frequency f_i when both waves induce the same voltage in the antenna. The steady-state selective coefficient of the simple series antenna circuit then is

$$S_c = \frac{P_c}{P_i} = \frac{Z_i^2}{Z_c^2} \tag{5}$$

If the circuit is resonant to the frequency of the correspondent station, then

$$Z_c = R_w + R_r + R_d - N = R_t - N = R_n \tag{6}$$

$$S_c = \frac{Z_i^2}{R_n^2} \tag{7}$$

In some discussions it is convenient to have the steady-state selective coefficient expressed in terms of the time constant of the circuit and the per cent detuning of the interferent station. Let the resonant angular velocity of

the circuit be ω_c and the angular velocity of the interferent waves be $\omega_c(1 + p_d)$. Then the reactance of the circuit to the interferent frequency is

$$X_i = \omega_c L(1 + p_d) - \frac{1}{(1 + p_d)\omega_c C}$$

But

$$\omega_c L = \frac{1}{\omega_c C}$$

Therefore

$$X_i = \omega_c L\left(1 + p_d - \frac{1}{(1 + p_d)}\right)$$

But

$$1 + p_d - \frac{1}{1 + p_d} = 1 + p_d - (1 - p_d + \cdots) = 2p_d \quad \text{(approximately)} \quad (8)$$

if p_d is small compared to unity. Therefore when the interferent frequency differs by only a few per cent from the correspondent frequency, the steady-state selective coefficient becomes

$$S_c = \frac{R_n^2 + 4\omega_c^2 P_d^2 L^2}{R_n^2} = 1 + \omega_c^2 p_d^2 \frac{4L^2}{R_n^2}$$

$$S_c = 1 + \omega_c^2 p_d^2 T_c^2 \quad (9)$$

In Eq. (9) T_c represents the time constant of the circuit.

Since the second term of Eq. (9) is in general large compared to unity, it is evident that the steady-state selective coefficient of a simple series circuit associated with a pure resistance neutralizer varies as the square of the time constant. Now

$$T_c^2 = \frac{4L^2}{(R_w + R_r + R_d - N)^2} = \frac{4L^2}{(R_t - N)^2} \quad (10)$$

Equations (9) and (10) bring out in a striking manner the increase in the steady-state selective coefficient of a simple series circuit which may be obtained by associating a pure resistance neutralizer with it. Thus if $R = 100$ and $N = 99$ ohms, the presence of the neutralizer increases the

steady-state selective coefficient by the factor $(100)^2 =$ 10,000.

In actual communication systems we are not dealing with steady-state phenomena, and it will be shown in later sections that there is an upper limit for T_c which cannot be exceeded and still have satisfactory reception of signals. A detailed discussion of this point is deferred to later sections. It can be stated, however, that for many classes of signals this limit for T_c is so high that it is hard to obtain without resistance neutralization. In these cases the resistance neutralizer can be used to great advantage in increasing the selective properties of the simple series receiving circuit.

It has been shown in Chap. III that the regenerative amplification at resonance due to a resistance neutralizer is

$$\text{regenerative amplification} = \frac{R_t^2}{(R_t - N)^2} \qquad (11)$$

If $R_t = 100$ ohms and $N = 99$, the presence of the neutralizer increases the power due to the correspondent station by the factor 10,000. The theory of Chap. III shows that the amplification of power from dissonant frequencies is nearly unity. Equation (25) of Sec. 13 shows that the neutralizer would cause the waves of the correspondent station to deliver 100 times as much power as they would deliver if the neutralizer were not in the circuit. The neutralizer multiplies the power delivered by the correspondent waves by the factor 100, thus bringing the total power multiplication up to 10,000.

28. Application to Triode Circuits.

In Chap. III it was pointed out that triode circuits seldom function as pure resistance neutralizers. If the resistance neutralization, however, varies slowly with the frequency and if the reactance introduced into the circuit by the neutralizer is small, then the theory of pure resistance neutralization needs but little modification to be applicable to these circuits. As an example of a vacuum tube circuit which fulfils these conditions, we will take the circuit of Fig. 25. Let

the condenser be the antenna of a radio receiving circuit and let the generator represent the voltage induced in the antenna by the impinging waves. The following constants will be assumed for the tube and circuit:

$G_{cp} = 1{,}025 \times 10^{-6}$ mhos	$R_p = 40$ ohms
$G_p = 350 \times 10^{-6}$ mhos	$L_g = 10^{-3}$ henrys
$L_1 = 10^{-3}$ henrys	$M_p = 10^{-4}$ henrys
$R_1 = 61$ ohms	$M_g = -2 \times 10^{-4}$ henrys
$L_p = 5 \times 10^{-4}$ henrys	$\lambda_c = 943$ meters
	$\omega_c = 2 \times 10^6$ radians per second

Let the interferent station be detuned by 3 per cent. The equations for the circuit under consideration are given in Sec. 11. Values at resonance:

$$\omega = \omega_c = 2 \times 10^6$$

$$h = M_p(-M_g G_{cp} - M_p G_p)$$

$$= 10^{-4}[(1{,}025 \times 10^{-6})(2)10^{-4} - (350)10^{-10}] = 1.7 \times 10^{-11}$$

$$D = 1 + R_p G_p = 1.014$$

$$\omega_c^2 L_p^2 G_p^2 = 0.1225$$

$$X_c = \frac{\omega_c^2 h D}{D^2 + \omega_c^2 L_p^2 G_p^2} = 59.93 \text{ ohms}$$

$$X_{ac} = \frac{\omega_c^3 L_p G_p h}{D^2 + \omega_c^2 L_p^2 G_p^2} = 20.7 \text{ ohms}$$

$$\omega_c L_1 = 2(10^6)10^{-3} = 2 \times 10^3$$

If the reactance of the circuit is to be zero to the correspondent station, then

$$\omega_c L_1 - \frac{1}{\omega_c C} + 20.7 \text{ must equal zero}$$

$$C = 2.474 \times 10^{-10} \text{ farads}$$

The power delivered to the detector at resonance is

$$P_c = \frac{E^2}{(61 - 59.93)^2} R_d = \frac{E^2 R_d}{1.145}$$

Values off resonance:

$$\omega = \omega_c - 0.03\omega_c$$
$$\omega_i^2 L_p^2 G_p^2 = 0.115$$
$$N_i = 56.33$$
$$X_{A_i} = 18.85$$
$$X_i = \omega_i L_1 - \frac{1}{\omega_i C} + 18.85 = 124.7$$
$$Z_i = \sqrt{(R_t - N)^2 + X_i^2} = 124.7$$

The power delivered to the detector due to the interferent waves is

$$P_i = \frac{E^2 R_d}{(124.7)^2}$$

The steady state selective coefficient is

$$S_c = \frac{P_c}{P_i} = \frac{(124.7)^2}{1.145} = 1.35 \times 10^4$$

The regenerative amplification is

$$\text{regenerative amplification} = \left(\frac{R_t}{R_t - N}\right)^2 = \left(\frac{61}{1.07}\right)^2 = 3.25 \times 10^3$$

The steady-state selective coefficient without the tube to a station detuned by 3 per cent would be, from Eq. (9),

$$S_c = 1 + (4 \times 10^{12})(3 \times 10^{-2})^2 \frac{(4)10^{-6}}{(61)^2} = 5.3$$

The triode thus increases the steady-state selective coefficient against a station detuned 3 per cent by the factor 2,559.

29. Effect of Resistance Neutralization upon the Power Which Can Be Abstracted by an Antenna from Impinging Waves.

If resistance neutralization is not resorted to, the value of the detector resistance which will make the delivery of power from sustained waves to the dectector a maximum is

$$R_d = R_r + R_w \tag{12}$$

where R_d represents the equivalent series detector resistance, R_r the radiation resistance of the antenna with which

the detector is associated, and R_w the wasteful resistance of the antenna circuit. Let us seek the expressions for the optimum values of R_d with resistance neutralization under the conditions stated in the following problems.

Problem 1.—A given antenna has at a given frequency a given radiation resistance R_r and a given wasteful resistance R_w. It is desired to deliver the maximum possible power to a utilization device (detector) having an undetermined resistance R_d. A resistance neutralizer is available which will operate reliably (steadily) to reduce the total resistance R_t (or $R_d + R_w + R_r$) to a net resistance R_n, which is γ decimal parts of the total resistance; that is, by the use of the neutralizer

$$R_n = \gamma(R_d + R_r + R_w) \tag{13}$$

What value should be assigned to the detector resistance R_d to make the power delivered to it a maximum when waves of resonant frequency impinge upon the antenna?

The antenna current caused by an electromotive force of r.m.s. value E of resonant frequency is

$$I = \frac{E}{R_n} = \frac{E}{\gamma(R_d + R_r + R_w)}$$

The power expended in the detector is

$$P_t = I^2R_d = \frac{E^2R_d}{\gamma^2(R_d + R_r + R_w)^2}$$

The value of R_d which makes the power P a maximum, as found by equating the derivative of P with respect to R_d to zero and solving, is

$$R_d = R_r + R_w \tag{12}$$

If R_d has this value,

$$R_n = 2\gamma(R_r + R_w) \tag{14}$$

and

$$P_T = \frac{E^2}{4\gamma^2(R_r + R_w)} \tag{15}$$

Problem 2.—In the problem above, no lower limit was placed upon the value of the net resistance, and in satis-

fying the conditions for maximum power the net resistance was reduced to $2\gamma(R_r + R_w)$. But suppose this low net resistance results in a circuit time constant which is of prohibitive length. In other words, let it be assumed that it is not permissible to reduce the net resistance R_n below a specified value R_m. Under these limiting conditions the value assigned to R_d must be such as to make R_n or $\gamma(R_d + R_r + R_w)$ not less than R_m. That is, R_d must not be less than (but may be greater than)

$$\frac{R_m}{\gamma} - (R_r + R_w)$$

If $\left[\frac{R_m}{\gamma} - (R_r + R_w)\right] > (R_r + R_w)$, or if $\frac{R_m}{\gamma} > 2(R_r + R_m)$, the value which must be assigned to R_d in order to limit the net resistance R_m (or the time constant) as specified above is as follows:

$$R_d = \frac{R_m}{\gamma} - (R_r + R_w) \tag{16}$$

This is greater than the value for maximum power delivery.

On the other hand, if $\frac{R_m}{\gamma} \leqslant 2(R_r + R_w)$, the value to be assigned to R_d is the optimum value specified in Eq. (12). If $\frac{R_m}{\gamma} = 2(R_r + R_w)$, this optimum value makes the net resistance R_n just equal to the lower limit R_m for the net resistance, or gives to the time constant the maximum permissible value.

If $\frac{R_m}{\gamma} < 2(R_r + R_w)$, this optimum value makes the net resistance R_n greater than the lower limit R_m, or has the effect of making the time-constant shorter than the maximum permissible value.

Problem 3.—Now suppose the problem is not that of making the power delivery to the detector a maximum, but the problem is to make the selective coefficient against a sustained wave detuned station a maximum. What is the

We limit the discussion to the general case in which the interferent station is sufficiently dissonant (2 to 5 per cent) to make the net reactance of the antenna to the dissonant frequency large in comparison with its net resistance. From Eq. (9) it is seen that the selective coefficient of the antenna against an interferent electromotive force which is detuned by a given percentage from the given resonant frequency of the antenna is substantially proportional to the square of the time-constant (T_c) of the antenna. The selective coefficient is independent of the value of the detector resistance, except as the detector resistance may affect the value of the time-constant. It then we are dealing with an antenna of given height and capacity, the time-constant of which may not be permitted to exceed a specified value, such as 0.01 second, two cases arise:

Case I.—If the sum of the radiation and the wasteful resistance of the given antenna is so large that $\gamma(R_r + R_w)$ by itself is greater than the resistance R_m which corresponds to the maximum permissible time-constant, then the maximum selective coefficient will be obtained if the detector resistance is allowed to approach zero. The power delivered to the detector at the resonant frequency, however, is a maximum when $R_d = (R_r + R_w)$, and the power decreases to zero as R_d approaches zero.

Case II.—If $\gamma(R_r + R_w)$ is less than R_m, the value of the selective coefficient is fixed by the value assigned to T_c (or to R_m), and is independent of the value assigned to R_d, provided that $\gamma(R_d + R_r + R_u)$ is made equal to R_m.

Before the advent of the resistance neutralizer all antennae fell under Case I. By the proper use of neutralizers all antennae may be made to fall under Case II. The question which now arises is this: If the maximum selective coefficient possible by the use of a single tuned circuit has been obtained by satisfying the relation

$$\gamma(R_d + R_r + R_u) = R_m \qquad (17)$$

in which R_m is the resistance which gives the maximum permissible time-constant, what further conditions should

be satisfied to make the power delivered to the detector resistance at the resonant frequency a maximum? Two subcases arise under this Case II.

Subcase A.—In this case we have a **given** antenna whose dimensions are not to be changed. The only thing which may be varied is the detector resistance R_d. Since in this case the values both of R_m and of the selective coefficient are fixed by the assignment of a value to the time-constant, and since, at resonance, R_m alone determines the flow of current per volt induced in the antenna, and since the power delivered to the detector is

$$P = I^2 R_d = \left(\frac{E}{R_m}\right)^2 R_d \tag{18}$$

we may formulate the following rule:

To obtain from a **given** antenna the maximum power consistent with a specified time-constant (or selective coefficient), the detector resistance R_d should be made as great as possible consistent with the stable reduction of the total resistance to the net value R_m which is fixed by the specified time-constant.

Subcase B.—In this case the problem is to determine the proportions which the antenna itself should have in order to deliver the maximum power (consistent with the specified time-constant) to a detector when the antenna is used with a neutralizer having a fixed resistance reduction factor γ.

Let C_0 = capacity of the antenna
L_0 = inductance of the antenna circuit
f_r = frequency of the correspondent station
T_c = desired time-constant
R_m = net resistance for the specified time-constant
h = height of the antenna network in centimeters
s = velocity of light, 3×10^{10} centimeters per second
p_0 = permittivity of air 8.85×10^{-14} farad-centimeters
F_m = peak value of the electric intensity at the antenna in volts per centimeter

The expression for the power delivered to the detector is

$$P = \frac{(F_m h)^2 R_d}{2R_m^2} \tag{19}$$

To obtain the maximum selective coefficient the value assigned to R_d must satisfy Eq. (16); that is, R_d must equal

$$\frac{R_m}{\gamma} - (R_r + R_w) \tag{16}$$

In the subsequent discussion the value of the ratio

$$\frac{R_r + R_w}{R_r} \text{ will be represented by } k \tag{20}$$

and k will be treated as a constant. It should be recognized that this is not strictly correct but is an approximation only.

Substituting the value of R_d from Eqs. (16) and (20) in the equation for the power, it becomes

$$P = \frac{F_m^2 h^2}{2R_m^2}\left(\frac{R_m}{\gamma} - kR_r\right) \tag{21}$$

An expression for the value of the minimum permissible net resistance R_m in terms of the antenna constants and specified time-constant may be arrived at as follows:

$$f_r = \frac{1}{2\pi\sqrt{L_0 C_0}} \text{ or } L_0 = \frac{1}{4\pi^2 f_r^2 C_0}$$

$$T_c = \frac{2L_0}{R_m} = \frac{1}{2\pi^2 f_r^2 C_0 R_m}$$

from which

$$R_m = \frac{1}{2\pi^2 f_r^2 C_0 T_c} \tag{22}$$

That is, the value of R_m is fixed by T_c, C_0, and f_r.

In any antenna with an extended capacity area at a height h, the expression for the radiation resistance may be written

$$R_r = \frac{160\pi^2 h^2}{\lambda^2} = \frac{4\pi h^2 f_r^2}{3s^3 p_0} \tag{23}$$

Substituting the values of R_m and R_r as expressed in Eqs. (22) and (23) in Eq. (21), we have the following equation for the power delivered to the detector when the antenna capacity, the detector resistance, and the reduction factor γ are so related as to give the specified selective coefficient:

$$P = (F_m h)^2 2\pi^4 f_r^4 C_0^2 T_c^2 \left[\frac{1}{2\pi^2 f_r^2 C_0 T_c \gamma} - \frac{4k\pi h^2 f_r^2}{3s^3 p_0} \right] \qquad (24)$$

If the radius of the antenna network is so great as compared with the mounting height that the capacity is approximately expressed by the parallel plate formula, namely,

$$C_0 = \frac{p_0 a}{h} \qquad (25)$$

the following equation results from the substitution of the value of C_0 from Eq. (25) in Eq. (24):

$$P = F_m^2 2\pi f_r^4 p_0^2 a^2 T_c^2 \left[\frac{h}{2\pi^2 f_r^2 p_0 a \gamma T_c} - \frac{4k\pi h^2 f_r^2}{3s^3 p_0} \right] \qquad (25a)$$

To find the antenna height or the antenna area which will make the power a maximum, we take the derivatives of P with respect to h or to a respectively, equate the derivatives to zero, and solve the resulting equations. Upon doing so, it is found that the antenna should be so proportioned that

$$\frac{h}{2\pi f_r^2 p_0 a \gamma T_c} = \frac{8k\pi h^2 f_r^2}{3s^3 p_0} \qquad (26)$$

That is, the values assigned to h or to a must be such that

$$\frac{R_m}{\gamma} = 2(kR_r) \qquad (27)$$

in which case

$$R_d \text{ will equal } kR_r \text{ or } R_r + R_w \qquad (28)$$

Equation (26) may also be written in the form

$$ah = \frac{3s^3}{16\pi^3 k \gamma T_c f_r^4} \qquad (29)$$

On the other hand if the antenna network is so high that its capacity is approximately expressed by the formula for an elevated circular disk, namely,

$$C_0 = 8p_0\sqrt{\frac{a}{\pi}} \tag{30}$$

the following equation results from the substitution of this value of C_0 in Eq. (24):

$$P = (F_m^2h^2)128p_0^2\pi^3f_r^4aT_c^2\left[\frac{1}{16\pi^{3/2}f_r^2p_0a^{1/2}\gamma T_c} - \frac{4k\pi h^2f_r^2}{3s^3p_0}\right]$$

Upon taking derivatives of this value of P with respect to h and a, equating to zero and solving, it is found that the antenna should be so proportioned that

$$\frac{1}{16\pi^{3/2}f_r^2p_0a^{1/2}\gamma T_c} = \frac{8k\pi h^2f_r^2}{3s^3p_0} \tag{31}$$

That is, in this case also the values assigned to h and to a must be such that

$$\frac{R_m}{\gamma} = 2(kR_r) \tag{27}$$

Equation (31) may also be written in the form

$$a^{1/2}h = \frac{3s^3}{128\pi^{5/2}k\gamma T_cf_r^4} \tag{32}$$

Equations (29) and (32) give respectively the dimensions which antennae of the parallel-plate type and the elevated-disk type must have to permit of the maximum power delivery to the detector, and the maximum selective coefficient against detuned frequencies, which is possible with the given resistance ratio k, reduction factor γ, time-constant T_c, and frequency f_r. These equations express, not exactly, but only approximately, the optimum relations between the antenna dimensions and the four quantities k, γ, T_c, and f_r. They are valid only for antennae of the usual proportions found in high-power practice, that is, for antennae whose greatest length is short (one-eighth or less) in comparison with the wave length.

The total power delivered to the detector associated with a simple series antenna is

$$P_T = \frac{h^2F_m^2}{2}\frac{R_d}{R_m^2} \tag{19}$$

If

$$R_d = (R_r + R_w) = kR_r \tag{12}$$

Eq. (19) may be written

$$P_T = \frac{h^2F_m^2}{8\gamma^2 kR_r} \tag{33}$$

Of the total power P_T, the amount P_B abstracted from the impinging wave is

$$P_B = \frac{h^2F_m^2}{8\gamma kR_r} \tag{34}$$

and the amount P_A supplied by the neutralizer is

$$P_A = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma}\right)\frac{h^2F_m^2}{8kR_r} \tag{35}$$

By substituting the value of the radiation resistance from Eq. (23) in Eq. (34), the following expression is obtained for the power which is abstracted from the impinging waves and delivered to a detector resistance proportioned for maximum power as in Eq. (12):

$$P_B = \frac{3}{16\pi k\gamma}(s\lambda^2)(\tfrac{1}{2}p_0F_m^2) \tag{36}$$

In the paper referred to above, the factor $(s\lambda^2)(\frac{1}{2}p_0F_m^2)$ is shown to represent the power flowing across a **wave-length square** at the receiving station. Therefore the greatest power which can be delivered to a detector by an antenna from impinging waves is

$$\frac{3}{16\pi k\gamma}$$

times the power flowing across a **wave-length square** at the receiving station.

In this same paper the factor $\frac{1}{k}$ was termed the **abstractive factor** of the antenna. With a neutralizer associated

directly with the antenna the expression for the abstractive factor A_f of the antenna becomes

$$A_f = \frac{1}{k\gamma} \tag{37}$$

Equation (37) shows that the power abstractive factor of any existing antenna can be increased by associating with the antenna a resistance neutralizer, but it should be remembered that the increase in abstractive factor is accompanied by an increase in the time-constant of the antenna. If then the antenna circuit without the neutralizer has the longest time-constant which is permissible at the sending speed (for example 0.01 second at 30 words per minute), increased power from the waves can be obtained only by increasing the dimensions of the antenna. This may readily be seen by examining the expression giving the proportions which an antenna of the parallel-plate type must have for maximum selective coefficient, namely, Eq. (29).

$$ah = \frac{3s^3}{16\pi^3 k\gamma T_c f_r^4} \tag{29}$$

From this it is seen that if k and f_r are fixed, and if T_c is to remain constant, the volume under the antenna must be proportional to the reciprocal of the reduction factor of the neutralizer.

30. Introduction to the Method to Be Used in Arriving at the Actual Behavior of Receiving Circuits to Signals and to Interference.

The behavior of a radio receiving system to signals and to interference depends upon the transient state properties of the system. It is often a difficult problem to arrive at the transient-state properties of a system, whereas the formulation of the steady-state properties is a relatively easy matter. It is the purpose of the next few sections of this chapter to develop a relatively simple but effective way of answering some of the questions which arise in dealing with the effects of both signals and interference upon radio receiving systems when the steady-state properties of the

system are known. It is the purpose of this section to establish a viewpoint which will be an aid in obtaining an understanding of the following sections.

Figure 42 shows an alternator delivering a sinusoidal voltage and feeding a transformer on open circuit through a long line A. Parallel to this line there is another line B which may be a telephone circuit. We wish to find the

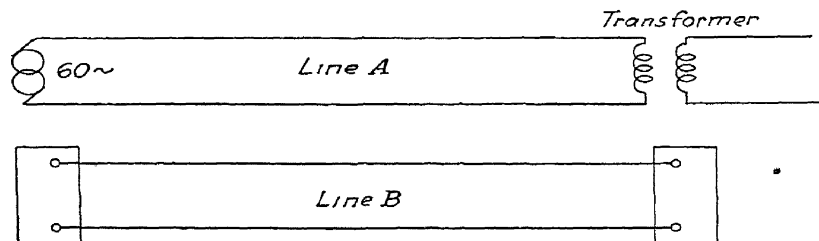



FIG 42.

effect of line A upon line B. We will suppose that line A is a low-tension line so that the electrostatic effects can be neglected. The current in line A is the magnetizing current of the transformer and therefore this current has the form shown by Fig. 43. This current can be broken up into the two sine waves shown by Fig. 44 and a number of smaller

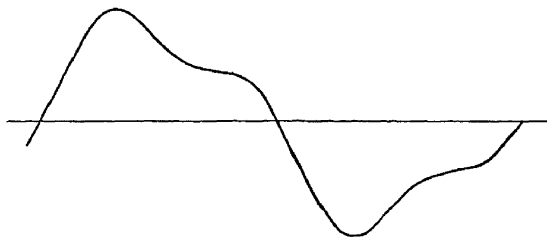



FIG. 43

components of higher frequencies which have not been shown. One of these sine waves has a frequency of 60 cycles per second and an amplitude I. The other has a frequency of 180 cycles per second and an amplitude of $\frac{I}{4}$. If the mutual inductance between the lines is represented by M, then the voltage induced in line B is composed of the following two parts. Part one is a sine voltage having a

frequency of 60 cycles per second and an amplitude of $2\pi 60IM$ volts. Part two is a sine wave having a frequency of 180 cycles per second and an amplitude of $2\pi 180M\frac{I}{4}$. If the amplitude of the 60-cycle voltage is represented by E, then the amplitude of the 180-cycle voltage is $\frac{3}{4}E$. We can forget all about line A then, if in line B we introduce two alternators. One alternator has a frequency of 60

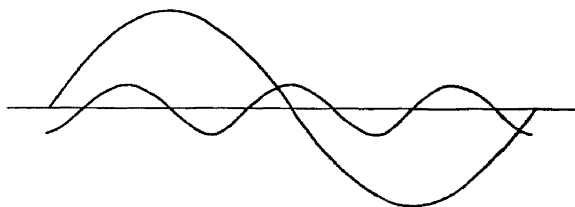



FIG. 44.

cycles per second and delivers a voltage of amplitude E. The other alternator has a frequency of 180 cycles per second and a voltage of amplitude $\frac{3}{4}E$. These generators must have zero impedance. The effects of line A upon line B can be calculated by considering the system shown by Fig. 45.

The scheme used above for finding the effect of line A on line B is the one used in the following sections of this chapter

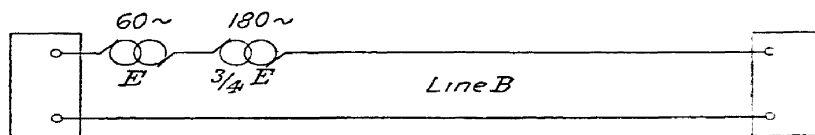



FIG 45

for finding the manner in which a voltage induced in the antenna affects the receiving system. The voltage induced in the antenna is replaced by a group of alternators having the correct voltages and the correct frequencies. The receiving system will then be represented schematically as shown by Fig. 46. The generators are assumed to have no impedance and serve only as a device to fix the attention on a sine wave of voltage having a given frequency and

amplitude. Use is made of Fourier's expansion in order to obtain the frequency and voltage of each alternator. Since the time interval in the expansions will be taken from minus infinity to plus infinity, each of the alternators will have been in the circuit for an infinitely long time. From this fact it follows that the response of the system to transient electromotive forces may be arrived at from a knowledge of the steady-state properties of the system.

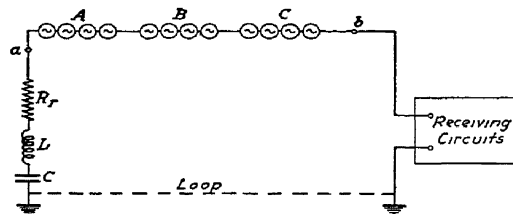



FIG 46.—Schematic diagram illustrating the replacement of voltages induced in a receiving antenna by alternators.

31. The Generators Which Replace the Voltage Induced in a Receiving Antenna by an Interrupted Continuous-wave Transmitting Station.

The voltage induced in a receiving antenna by an interrupted continuous-wave telegraph station is assumed to have the form shown schematically by Fig. 47. In this

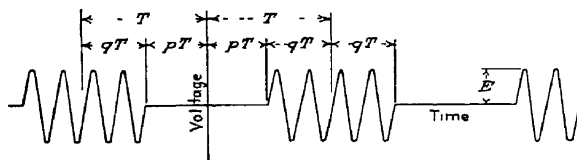



FIG. 47.—Voltage assumed to be induced in a receiving antenna by an I C W transmitting station.

figure $2T$ represents the total time interval of the signal and the following space; $2qT$ represents the signal time interval; and $2pT$ represents the space time interval. Thus $p + q$ are two such fractions that their sum is always unity. In order to simplify the calculations it will be assumed that there are a complete number of cycles of the operating frequency in the time intervals qT and pT. Under these conditions if the operating frequency is represented by

$f_0 = \frac{\omega}{2\pi}$, the voltage induced in the antenna is represented from

$$t = -\infty \text{ to } t = +\infty$$

by the series

$$e(t) = \sum_{m=1}^{\infty} a_m \sin \frac{m\pi t}{T}^* \tag{38}$$

in which

$$a_m = \frac{E}{T}(\sin mp\pi)\left[\frac{2\omega}{\omega^2 - \frac{m^2\pi^2}{T^2}}\right]^* \text{ for } \frac{m\pi}{T} \neq \omega \tag{39}$$

$$= Eq \text{ for } \frac{m\pi}{T} = \omega \tag{40}$$

That is, the voltage of the mth generator is given by Eqs. (39) and (40). The generator having the largest voltage is the one having a frequency the same as the operating frequency f_0 of the transmitting station. The frequency of the mth generator is, from Eq. (38),

$$f_m = \frac{1}{2\pi}\left(\frac{m\pi}{T}\right) = \frac{m}{2T} \tag{41}$$

That is, the generators in the receiving circuit which replace the induced voltage are spaced $\frac{1}{2T}$ cycles apart. The value of T depends upon the speed of signaling. At 30 words per minute, $T = 0.05$ second. At 150 words per minute $T = 0.01$ second. Thus the speed of signal transmission determines the frequency spacing of the generators.

Let m_0 represent the m which gives the operating frequency, f_0.

Then

$$m_0 = 2Tf_0 \tag{42}$$

Let

$$m = m_0 + n = 2Tf_0 + n \tag{43}$$

$$n = 0, \pm 1, \pm 2, \pm 3 \cdots \pm^{\infty}_{-(2Tf-1)}$$

* For the derivation of these equations see Appendix A, Part I.

If the signal sent out is a Morse dot and the interval between signals is the Morse dot interval, then

$$p = q = 0.5 \tag{44}$$

In Eq. (39)

$$\sin mp\pi = \sin m\frac{\pi}{2} = 0 \text{ if } m \text{ is even}$$
$$= \pm 1 \text{ if } m \text{ is odd}$$

Upon substituting these results and Eq. (43) in Eq.(39) and remembering that $\omega = 2\pi f$, we obtain

$$a_{m_0} = \frac{E}{2} \tag{45}$$

$$\left|a_{m_0+n}\right| = \frac{E}{n\pi}\left[\frac{1}{1 + \frac{n}{4Tf_0}}\right] \tag{46}$$

From Eq. (42) it is evident that m_0 will always be an even number. It follows, then, since $m_0 + n$ must be odd, that n will be an odd number. Thus we write

$$n = 0, \pm 1, \pm 3, \pm 5 \cdot \cdot \cdot {}^{+\infty}{}_{-(2Tf_0 - 1)} \tag{47}$$

For all of the important frequencies[1] $\frac{n}{4Tf_0}$ is small compared to unity, and Eq. (46) may be simplified to

$$\left|a_{m_0+n}\right| = \frac{E}{n\pi} \tag{48}$$

We have now arrived at the following information relative to the generators which replace the induced voltage in the receiving antenna. The generator having the highest voltage is the one having a frequency equal to the operating frequency f_0. The voltage of this generator is $\frac{E}{2}$. As we pass to generators having a frequency less than f_0, the first one we come to has a frequency $f_0 - \frac{1}{2T}$. This generator has a voltage of $\frac{E}{\pi}$. The next has a frequency of $f_0 - \frac{3}{2T}$

[1] See Appendix A, Part I.

and has a voltage of $\frac{E}{3\pi}$ and so on down the line. The first generator having a frequency higher than f_0 is the one which has a frequency $f_0 + \frac{1}{2T}$. The voltage of this generator is also $\frac{E}{\pi}$. The next one up the frequency scale has a frequency equal to $f_0 + \frac{3}{2T}$ and has a voltage equal to $\frac{E}{3\pi}$ and so on. These facts are brought out in a striking manner by the curve of Fig. 48. This curve is plotted with the ratios of the absolute value of the generator voltage to E as ordinates and with values of n as abscissas. Values of n are used as abscissas in order to make the curve hold for all speeds of transmission. To convert the abscissa scale to generator frequencies, use is made of the relation

$$\text{Generator frequencies} = f_0 + \frac{n}{2T} \tag{49}$$

At 30 words per minute this relation becomes

$$\text{Generator frequencies} = f_0 + 10n \tag{50}$$

At 150 words per minute it becomes

$$\text{Generator frequencies} = f_0 + 50n \tag{51}$$

Thus a generator having a given ratio of voltage to E is five times as far removed in frequency from the operating frequency at a transmitting speed of 150 words per minute as at a transmitting speed of 30 words per minute. This fact, as we shall see later, has an important bearing upon the design of the receiving system and also upon the amount of interference created by the transmitting station.

The desirable frequency-response characteristics of the interrupted continuous-wave receiving system can now be arrived at. If the system were to pass currents having the operating frequency f_0, and were to eliminate currents of all other frequencies, a continuous tone would be heard in the receivers, and the dots and spaces could not be distinguished from each other; that is, no signals would be

received. If the system passed currents of all frequencies with the same ease, the high-frequency output would have

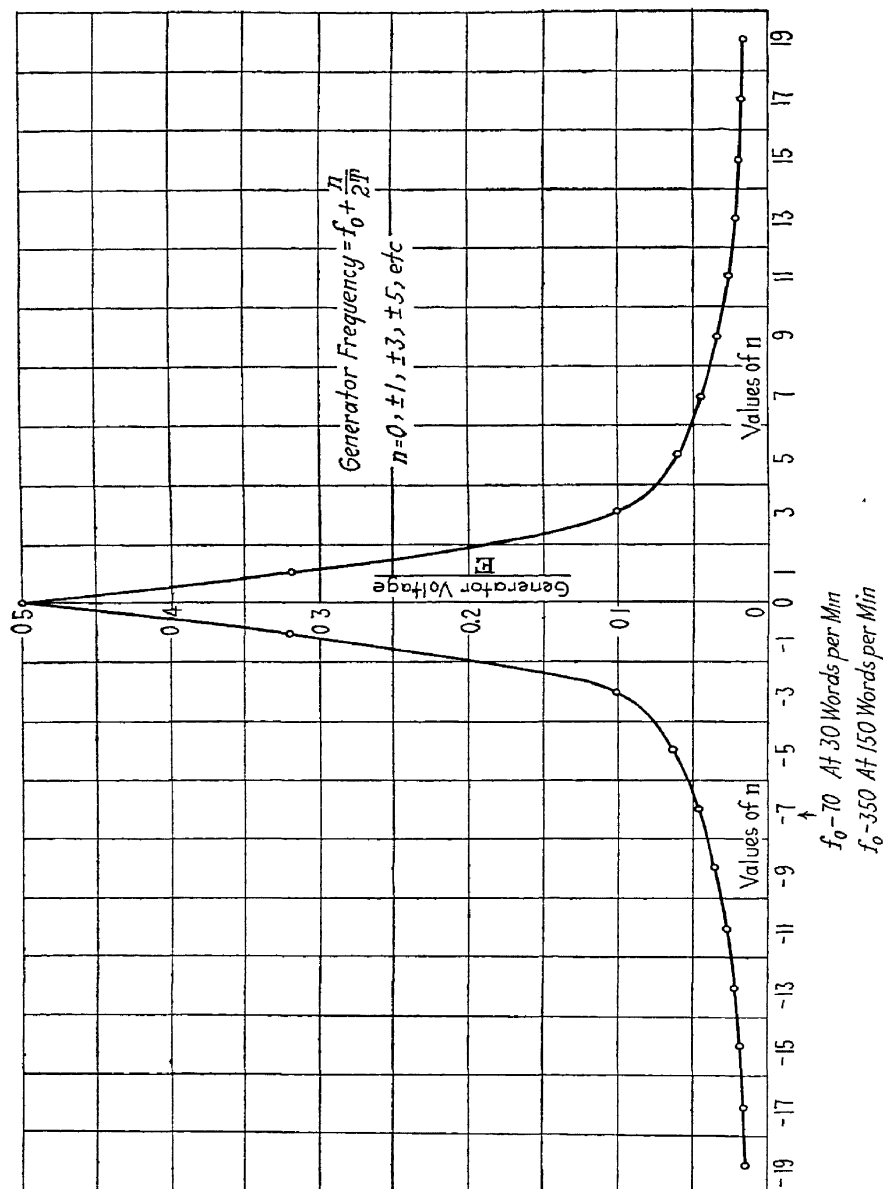



FIG 48.—Valuation of generator voltage with frequency in I C.W telegraphy

the same wave form as the induced voltage (Fig. 47). This latter condition would lead to the distinguishing of the signals, but the system would have no selectivity against

interference. The best circuit then would be one which passed just enough frequencies to make the signals distinguishable and eliminated all others. Let us assume that in order to make the signals distinguishable, the receiving system must respond freely to all generators having a voltage greater than or equal to τ decimal parts of the voltage of the generator whose frequency is f_0. That is, the system must respond freely to all generators having a voltage equal to or greater than $\frac{\tau E}{2}$. Now the generator whose frequency is $f_0 \pm \frac{n}{2T}$ has a voltage equal to $\frac{E}{n\pi}$. We therefore write

$$\left(\frac{E}{n_\tau \pi}\right)\frac{2}{E} = \tau$$

$$n_\tau \text{ is the odd number closest to } \frac{2}{\tau\pi} \qquad (52)$$

The receiving system, therefore, must pass a band of frequencies $\frac{n_\tau}{T}$ wide centered on the frequency f_0.

Let us define an ideal receiving system as one which has the following properties:

1. A radiation resistance R_r.
2. No wasteful resistance.
3. It acts as a pure resistance of magnitude $2R_r$ to frequencies lying in the range $f_0 + \frac{n_\tau}{2T}$.
4. Currents having frequencies lying in the range shall be passed on to the detector either without attenuation or else with a uniform amplification.
5. Currents of all other frequencies shall be eliminated before reaching the detector.

A system having the above properties is called an ideal system because it represents the best possible frequency-selection system for receiving I.C.W. signals through interference. This ideal system can be approximated only more or less imperfectly in practice; but any actual receiving system, basing its selectivity upon frequency selection should be made to fulfil as closely as possible the conditions stated.

The band of frequencies which the receiving system must pass freely is $\frac{n_r}{T}$. Now both n_r and T are independent of f_0. So the band width is independent of the operating wave length. n_r is also independent of the speed of signal transmission while T varies inversely as the speed of transmission. Therefore the frequency band which the receiving system must transmit freely varies directly with the sending speed. Thus the transmitted band width at 150 words per minute would be five times as great as at 30 words per minute.

If the receiving system acted as a pure resistance of magnitude $2R_r$ to all frequencies, then during the dot interval the r.m.s. current would be

$$I_d = \frac{E}{2\sqrt{2}R_r} \tag{53}$$

and the average rate of useful power delivery would be

$$P_d = \frac{E^2 R_r}{8R_r^2} = \frac{E^2}{8R_r} \tag{54}$$

During the space interval the power delivery would be zero. Since we have taken the space interval equal to the dot interval, the average useful power delivery will be one-half of that given by Eq. (54), or

$$P_a = \frac{E^2}{16R_r} \tag{55}$$

In the ideal receiving system the mean-square current in the frequencies which are freely transmitted is

$$I^2 = \frac{E^2}{2}\frac{1}{4R_r^2}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \tag{56}$$

$$s = 1, 3, 5, \ldots \quad n_r$$

The average useful power delivered by the generators in the band of frequencies $f_0 \pm \frac{n_r}{2T}$ is

$$P = I^2 R_r = \frac{E^2}{8R_r}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \tag{57}$$

The ratio of the power actually utilized by the ideal receiver to the power available in the waves is therefore

$$\frac{P}{P_a} = 2\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \tag{58}$$

Calculations indicate that if $n_r = 3$, the intervals and the dots will be easily distinguishable. Under these conditions the ratio of the power utilized in the ideal circuit to the power available becomes

$$\frac{P}{P_a} = 2\left[\frac{1}{4} + \frac{2}{\pi^2}\left(1 + \frac{1}{9}\right)\right] = 0.9 \tag{59}$$

With the above value of n_r the ideal system would pass freely a band of frequencies lying between $f_0 - 30$ cycles per second and $f_0 + 30$ cycles per second at 30 words per minute. At 150 words per minute the band passed would be between $f_0 - 150$ cycles per second and $f_0 + 150$ cycles per second.

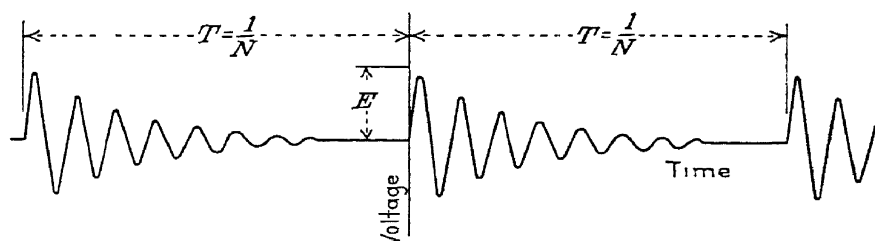



FIG. 49—Voltage assumed to be induced in a receiving antenna by a spark transmitter

32. Spark Telegraphy.

The voltage induced in a receiving antenna by a spark transmitting station is assumed to have the form shown schematically by Fig. 49. The operating frequency is $f_0 = \frac{\omega}{2\pi}$. In most spark systems the damping is such that the voltage dies very nearly to zero in the interval of time T when $\alpha T > 3$. Under these conditions the voltage is represented as a function of time during the interval $t = -T$ to $t = 0$ by the equation

$$e(t) = E\epsilon^{-\alpha(t+T)} \sin\left[\omega(t + T)\right] \tag{60}$$

and in the interval $t = 0$ to $t = +T$ by

$$e(t) = E\epsilon^{-\alpha t} \sin \omega t \tag{61}$$

The voltage induced in the antenna is represented from $t = -\infty$ to $t = +\infty$ by

$$e(t) = A_0 + \sum_{m=1}^{\infty} B_m \cos\left[\frac{m\pi t}{T} + \theta_m\right] \tag{62}$$

In our scheme of replacing the induced voltage by alternators we find from Eq. (62) that the voltage of the alternator whose frequency is $\frac{m}{2T}$ is equal to B_m.

It is shown in Appendix A, Part II, that B_m is given very closely by the equation

$$B_n = \frac{2E}{\omega T} \frac{1}{\sqrt{\frac{4\alpha^2}{\omega^2} + \left[\frac{2n}{f_0 T} + \frac{n^2}{f_0^2 T^2}\right]^2}} \tag{63}$$

$$n = 0, \pm 1, \pm 2 \; . \; . \; . \; {}_{-f_0 T}^{+\infty}$$

In this equation the symbols have the following meaning:

B_n is the voltage of the generator whose frequency differs from f_0 by $\frac{n}{T}$ cycles per second.

f_0 is the operating frequency of the spark transmitting station.

E and α are defined by Eq. (24).

$$\omega = 2\pi f_0$$

T is the time interval in seconds between sparks.

Let

$N = \frac{1}{T}$ represent the number of sparks per second

$\delta = \frac{\alpha}{f_0}$ represent the logarithmic decrement

In terms of these symbols Eq. (63) becomes

$$B_n = \frac{2NE}{\omega} \frac{1}{\sqrt{\frac{\delta^2}{\pi^2} + \left[\frac{2nN}{f_0} + \frac{n^2 N^2}{f_0^2}\right]^2}} \tag{64}$$

The voltage of the generator whose frequency is the same as the operating frequency f_0 is

$$B_0 = \frac{E}{\alpha T} = \frac{NE}{\alpha} = \frac{NE}{\delta f_0} \tag{65}$$

In order to present at a glance the way in which the generator voltage varies with the frequency in the vicinity of the operating frequency f_0 and the effect of the logarithmic decrement δ, the number of sparks per second N, and the operating frequency on this variation, the curves of Fig. 50 have been drawn. The abscissas for all curves are values of the difference between the generator frequency and the operating frequency $(= nN)$. Generator frequencies are located only at integral values of nN divided by N. Thus at 1,000 sparks per second one generator has the frequency f_0, and generators are located on a frequency scale every 1,000 cycles per second above or below f_0. At 120 sparks per second generators are located on the frequency scale at f_0 and every 120 cycles per second above or below f_0. For curve 1 $f_0 = 10^6$ cycles per second, $\delta = 0.01$, $\alpha = \delta f_0 = 10^4$, $N = 1{,}000$. The ordinates for this curve are values of the ratio of the generator voltage to the undamped peak voltage E_1, induced in the antenna. $\left(\text{Ordinates are values of } \frac{B_r}{E_1}.\right)$ The voltage assigned to the generators falls off at a fairly fast rate as the operating frequency is departed from. The generator having a frequency which differs by 16,000 cycles per second from the operating frequency has a voltage 10 per cent as great as the voltage of the generator whose frequency is equal to the operating frequency. That is, the energy associated with a frequency removed from the operating frequency by 16,000 cycles per second is 1 per cent as great as the energy associated with the operating frequency. This curve also holds good for the conditions $f_0 = 10^5$, $N = 1{,}000$, $\delta = 0.1$, $\alpha = \delta f_0 = 10^4$. This fact has an important bearing on the factors which determine the frequency-energy distribution, as will be shown a little later.

For curve 2, $f_0 = 10^6$, $N = 1{,}000$, $\delta = 0.1$, $\alpha = \delta f_0 = 10^5$. In order to make this curve directly comparable with curve 1, the ordinates have been taken as values of the generator voltage divided by the undamped voltage peak induced in

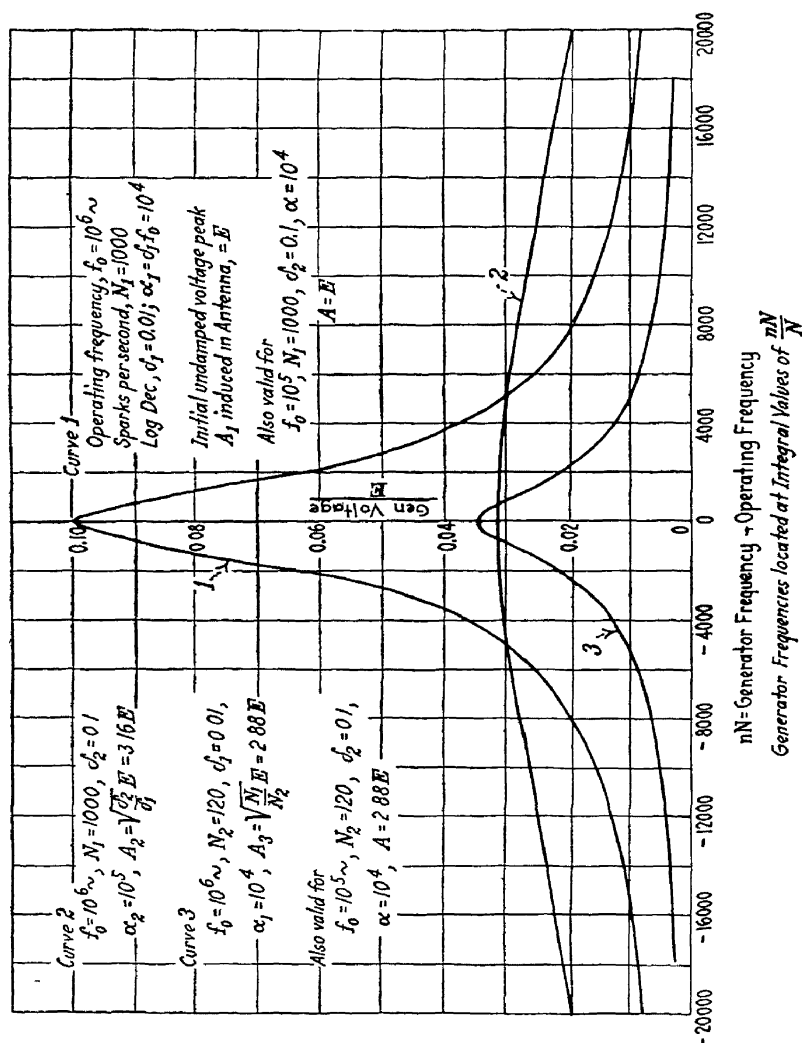



FIG 50 —Generator frequencies vs generator voltages in spark telegraphy

the antenna by case 1 when both sets of waves have the same energy per wave train or per spark. That is, the ordinates in this case are values of $\frac{B_n}{E_2}$ multiplied by $\sqrt{\frac{\alpha_2}{\alpha_1}}$.* This

* See the discussion of energy relations which follows shortly.

curve is much flatter than curve 1, and the generator whose frequency is removed from f_0 by 16,000 cycles per second has a voltage 70 per cent as great as the voltage of the generator whose frequency is f_0. This is a striking contrast to curve 1 for which $\alpha = 10^4$.

For curve 3, $f_0 = 10^6\sim$, $N_2 = 120$, $\delta = 0.01$, $\alpha = 10^4$. The ordinates for this curve are values of $\left(\frac{B_n}{E_3}\right)\sqrt{\frac{N_1}{N_2}}$.* This factor is used in order to compare this case with case 1 when both waves have the same energy associated with them. The rate at which the generator voltage falls off in this case with the frequency is about the same as for case 1. This curve is also valid for the conditions $f_0 = 10^5\sim$, $N = 120$, $\delta = 0.1$, $\alpha = 10^4$.

These curves lead to the conclusion that the factor which determines the rate at which the generator voltages fall off with the frequency in the vicinity of f_0 is $\alpha = \delta f_0$. The other factors have but little influence upon the width of the band of frequencies over which most of the energy is spread. This is apparent upon examining Eqs. (63) or (64). When n is small, the second term in the bracket under the radical is small compared to the first and may be dropped without serious error. We then have, for small values of n,

$$B_n = \frac{EN}{\sqrt{\alpha^2 + 4\pi^2 n^2 N^2}} \tag{66}$$

This equation shows clearly the dependence of the band width upon α, because the larger the value of α, the larger must be the nN product before B_n differs much from B_0.

The ideal system for receiving spark signals has the same properties as the ideal system for receiving I.C.W. signals, except that the transmitted band width will be different. In so far as obtaining a good tone in the receivers is concerned, it would suffice to pass only the currents of three generators. This would result, however, in the utilizing of only a small portion of the available energy. Let us therefore make the band wide enough to pass the currents

* See discussion of energy relations which follows shortly.

of all generators whose voltage is equal to or greater than τ decimal parts of the voltage of the generator whose frequency is f_0. From Eqs. (65) and (66) we then have

$$\frac{Bn_\tau}{B_0} = \frac{\alpha}{\sqrt{\alpha^2 + 4\pi^2 n_\tau^2 N^2}} = \tau \tag{67}$$

$$n_\tau N = f_c = \frac{\alpha}{2\pi}\sqrt{\frac{1}{\tau^2} - 1} \tag{68}$$

Since n_τ must be an integer, it is taken as the integer which comes the closest to satisfying Eq. (68). $n_\tau N = f_c$ is then one-half of the transmitted band width.

The mean square voltage induced in the receiving antenna by the impinging waves is, from Eq. (61),

$$E_a^2 = \frac{E^2}{T}\int_0^T [\epsilon^{-\alpha t} \sin \omega t]^2 dt$$

$$= \frac{E^2}{T}\left\{\frac{\epsilon^{-2\alpha t}}{4\omega^2 + 4\alpha^2}\left[(\sin \omega t)(-2\alpha \sin \omega t - 2\omega \cos \omega t) - \frac{\omega^2}{\alpha}\right]\right\}_0^T$$

Since $\epsilon^{-2\alpha t}$ is vanishingly small when $t = T$, we have

$$E_a^2 = \frac{E^2}{T}\left(\frac{1}{4\omega^2 + 4\alpha^2}\right)\frac{\omega^2}{\alpha} \tag{69}$$

If α^2 is small compared to ω^2, this reduces to

$$E_a^2 = \frac{E^2}{4\alpha T} = \frac{NE^2}{4\alpha} \tag{70}$$

If the receiving system acted as a pure resistance of magnitude $2R_r$ to all frequencies, the mean square current would be

$$I_a^2 = \frac{E_a^2}{4R_r^2} = \frac{NE^2}{4R_r^2 4\alpha}$$

The power delivered to a detector of resistance R_r would be

$$P_a = R_r I_a^2 = \frac{NE^2}{16\alpha R_r} \tag{71}$$

P_a as given by Eq. (71) represents the power available in the waves. Since this power varies directly as E^2, directly as N, and inversely as α, it is evident that the correction factors applied to the curves of Fig. 50 are the proper ones.

If E_n represents the r.m.s. value of the voltage of the generator whose frequency differs from f_0 by nN cycles per second, then the useful power delivered by this generator to the ideal receiver is 0 for n greater than n_τ and equals

$$P_n = \frac{E_n^2}{4R_r} \text{ for } n \leqq n_\tau \tag{72}$$

The total useful power delivered to the ideal circuit by the impinging waves is

$$P = \sum_n P_n = \frac{E_0^2}{4R_r} + 2\sum_{n=1}^{n_\tau} \frac{E_n^2}{4R_r} \tag{73}$$

The factor 2 enters the summation of Eq. (73) because there are two generators having the voltage E_n. One has a frequency of $f_0 + nN$ and the other has a frequency of $f_0 - nN$. Since the maximum value of n is n_τ, we may use Eq. (66) for finding the value of E_n^2. Since B_n is the peak value of the generator voltage $E_n^2 = \frac{1}{2}B_n^2$, Eq. (73) becomes

$$P = \frac{E^2N^2}{8R_r}\left[\frac{1}{\alpha^2} + 2\sum_{n=1}^{n_\tau} \frac{1}{\alpha^2 + 4\pi^2n^2N^2}\right] \tag{74}$$

The ratio of the power utilized by the ideal receiver to the power available is

$$\frac{P}{P_0} = 2\alpha N\left[\frac{1}{\alpha^2} + 2\sum_{n=1}^{n_\tau} \frac{1}{\alpha^2 + 4\pi^2n^2N^2}\right] \tag{75}$$

If the damping exponent α of the waves has a value of 10^4 and if τ is taken as 0.3, then the half band width as given by Eq. (68) is

$$f_c = n_\tau N = \frac{10^4}{2\pi}\sqrt{11.1 - 1} = 5{,}000 \sim \tag{76}$$

If the number N of sparks per second is 1,000, then $n_\tau = 5$, and the ratio of the power utilized by the ideal receiver to the power available is

$$\frac{P}{P_0} = (2)10^7\left[\frac{1}{10^8}(1 + 1.44 + 0.775 + 0.44 + 0.274 + 0.184)\right] = 0.82 \tag{77}$$

33. The Generators Which Replace the Voltage Induced in a Receiving Antenna by a Radio Telephone Transmitter. The voltage of the generators which may be inserted in the receiving system to replace the voltage induced in the antenna by a radio telephone transmitter cannot be written down in a general equation because these voltages depend upon the character of the speech or music which the transmitter is sending out. The theory of modulation which is developed in Chap. VII shows, however, that the voltage induced in the antenna of the receiving system by the telephone transmitter has a group of frequencies consisting of the carrier frequency, an upper side band, and a lower side band. The carrier frequency is the operating frequency and determines the wave length of the transmitting station. The upper side band consists of a group of frequencies having values equal to the carrier frequency plus the frequencies of the voice or musical notes. The lower side band consists of a group of frequencies having values equal to the carrier frequency minus the frequencies of the voice or musical notes. If the highest musical note of importance is represented by f_c, then in radio telephony most of the energy in the waves is associated with a band of frequencies $2f_c$ cycles wide centered on the carrier or operating frequency f_0. If only a small portion of the energy is associated with frequencies outside this band, then we may replace the voltage induced in the receiving system by a group of generators having frequencies ranging from $f_0 - f_c$ cycles per second to $f_0 + f_c$ cycles per second. The voltage assigned to a generator having a given frequency will depend upon the nature of the speech or music which is being received.

The ideal frequency-selection system for receiving telephony will have the same properties as the ideal system described for the reception of I.C.W. signals with the exception that it must transmit freely the currents due to all generators having frequencies in the band $f_0 - f_c$ to $f_0 + f_c$ cycles per second. This ideal system would utilize practically all of the available energy and would be distortion-

less. The highest musical note of much importance has a frequency of about 5,000 cycles per second. Therefore for broadcast reception, it suffices to assign the value 5,000 to f_c. From this it follows that the ideal receiver must transmit freely a band of frequencies 10,000 cycles wide centered on the operating frequency f_0.

34. Voltage Induced in a Receiving Antenna by Strays. The voltage induced in a receiving antenna by an I.C.W., a spark, or a telephone transmitter may be a source of interference as well as of signals. The frequency and voltage of the generators which may be used to replace the voltages due to these stations have been worked out. In order to make the discussion of interference more complete, we proceed to discuss the voltage and frequency of the

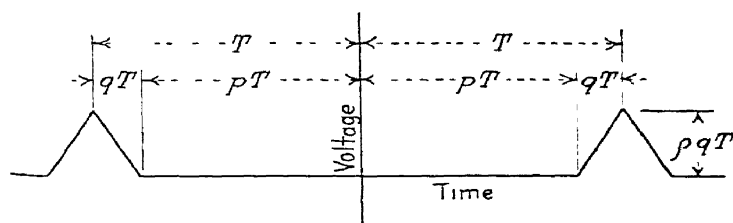



Fig 51 —Voltage assumed to be induced in a receiving antenna by strays

generators which may be used to replace the voltage induced in a receiving antenna by atmospheric strays. There is not very much information available on the wave form of strays. Watt and Appleton have published some observed wave forms of strays in the *Proceedings* of the Royal Society for 1923. These wave forms were sketched from visual observations made with a Braun tube oscillograph. The majority of the impulses were unidirectional in character and the time of rise and fall was about the same. The time duration of the majority of the impulses was of the order of $\frac{1}{1,000}$ second. It is reasonable to expect that some indication of the voltage and frequency of the generators which replace the static voltage induced in the receiving antenna by strays will be obtained if the wave form of the voltage induced in the antenna by atmospheric

strays is assumed to have the form shown by Fig. 51. It is shown in the Appendix, Part III, that this voltage can be represented from $t = -\infty$ to $t = +\infty$ by the series

$$e(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos \frac{m\pi t}{T} \tag{78}$$

where

$$a_0 = \rho T\,[\tfrac{1}{2} - p + \tfrac{1}{2}p^2] \tag{79}$$

$$a_m = \frac{2\rho T}{m^2\pi^2}\left[\cos m\pi - \cos m\pi p\right] \quad m \neq 0 \tag{80}$$

Equation (78) shows that the generators which replace the voltage of Fig. 51 have the frequency $\frac{m}{2T}$ and voltages given by Eqs. (79) and (80). Now the time duration of the impulses was around 10^{-3} seconds in the observations mentioned above. If the interval between impulses is assumed to be nine times as long, we have the relations

$$2qT = 10^{-3},\ 2pT = 9 \times 10^{-3}$$
$$T(2q + 2p) = 2T = 10^{-2}$$
$$T = 5 \times 10^{-3} \text{ seconds}$$

The frequency spacing of the generators then is $\frac{1}{2T} =$ 100 cycles. Equation (80) shows that the generators which have the high voltages have frequencies lying in the region from 100 to 1,000 cycles per second. That is, the high-voltage generators have frequencies far lower than the operating frequencies of any radio system. Since m enters the denominator of the expression for the generator voltages and since the m for a generator whose frequency is $2f$ is twice as great as for a generator whose frequency is f, it is evident that neglecting the periodic variation caused by the term in the brackets of Eq. (80), the voltage assigned to a generator is inversely proportional to the square of the frequency assigned to it. If we neglect the term in the brackets, then the voltage of all of the generators which have frequencies lying in a narrow band in the radio range is the same, because the m for the generator at one end of

the band differs only by a few per cent from the m for the generator at the other end of the band.

It is not safe to follow too closely the generator frequencies and voltages given by the wave form of Fig. 51 because of the meager information upon which it is based. The consideration of this wave form, however, enables us to draw certain general conclusions as to the frequency and voltage of the generators which replace the voltage induced in the receiving antenna by atmospheric strays. These inferences are as follows:

1. The voltage assigned to a generator having a high frequency is less than the voltage assigned to a generator having a lower frequency.
2. The voltage assigned to all of the generators having frequencies which lie in a narrow band in the radio range of frequencies will be about the same.

In regard to assumption 1 it may be stated that the wave form assumed indicated that the voltage assigned to a generator was inversely proportional to the square of the frequency assigned to it.

If a voltage wave form having an abrupt rise, a flat top, and an abrupt fall had been assumed, then the voltage assigned to a generator would be inversely proportional to the first power of the assigned frequency. Therefore, if the indications of the Watt and Appleton observations on the time duration of the impulses are approximately correct, the voltage assigned to a generator will vary inversely as a power of the frequency which lies between 1 and 2.

For the wave form assumed, assumption 2 holds good provided the periodic variation caused by the bracketed term of Eq. (80) is neglected. The justification for neglecting this term in drawing general conclusions arises from the fact that in the actual wave form of strays, the time duration of the impulses and the time interval between them vary at random. This random variation would smooth out the periodic variation indicated by the bracketed term of Eq. (80).

35. The Reception of the I.C.W. Signals through Interference Due to Atmospheric Strays.

The receiving system is represented schematically as shown in Fig. 46, and we now have to consider the power received from two groups of alternators. One group of alternators replaces the voltage induced in the antenna by the I.C.W. transmitting station. The other group replaces the voltage induced in the receiving antenna by strays. Since the voltage assigned to all the stray generators whose frequencies lie in a narrow band is the same, we will let E_s be the peak value of the voltage of each stray generator. The power delivered to the ideal receiving system by each generator whose frequency lies within the transmitted band is

$$P_{s1} = \frac{E_s^2}{8R_r} \qquad (81)$$

All other stray generators deliver no power to the detector. If the transmitted band of frequencies extends from $f_0 - f_c$ to $f_0 + f_c$, and if the stray generators are spaced p cycles apart on a frequency scale, and if f_0 is taken as the location of one of the generators, then the number of generators in the freely transmitted band is $\frac{2f_c}{p} + 1$. The power delivered by strays to the ideal receiving system is

$$P_s = \left(\frac{2f_c}{p} + 1\right)\frac{E_s^2}{8R_r} = \left(\frac{n_r}{pT} + 1\right)\frac{E_s^2}{8R_r} \qquad (82)$$

The power received from the I.C.W. transmitting station is given by Eq. (57). The ratio of signal power to stray power is

$$\frac{P}{P_s} = \frac{E^2}{E_s^2\left(\frac{n_r}{pT} + 1\right)}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \qquad (83)$$

$$S = 1, 3, 5, \ . \ . \ . \ n_r$$

Now because of the random nature of strays, one frequency is as likely to be the location of a generator as any other;

that is, p will be small compared to $\frac{n_r}{T}$. Under these conditions the ratio of signal power to stray power as given by Eq. (83) will be a maximum when the smallest possible value is assigned to n_r. In the ideal system n_r was taken just large enough to permit of the distinguishing of the signals. Therefore the ideal system already described is the best possible frequency-selecting system for receiving I.C.W. signals through stray interference. Since p is assumed small compared to $\frac{n_r}{T}$, the 1 may be dropped in Eqs. (82) and (83) compared to $\frac{n_r}{pT}$ and Eq. (83) may be written as

$$\frac{P_{I\,C.W}}{P_s} = \frac{E^2pT}{E_s^2}\frac{1}{n_r}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right]$$

$$= \frac{E^2p}{2E_s^2f_c}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \qquad (84)$$

In Eq. (84), $\frac{E_s^2}{p}$ characterizes the energy level of the strays associated with frequencies in the vicinity of the operating frequency f_0 of the I.C.W. station. This term decreases as f_0 increases. The term

$$E^2\left[\frac{1}{4} + \frac{2}{\pi^2}\sum \frac{1}{s^2}\right]$$

characterizes the energy level of the I.C.W. signal and its distribution over frequencies in the vicinity of f_0. The bracketed term is the same for all I.C.W. stations but E is dependent upon the transmitting station and the transmission efficiency. T characterizes the speed of signaling. It varies inversely with the signaling speed. Thus the high signaling speeds are more subject to static interference than the low ones. n_r alone characterizes the receiving system. For the ideal receiver, n_r has been assigned the minimum value which will permit of the distinguishing of the signals. Thus the ideal receiver reduces the static interference to the lowest possible value that can be obtained with a frequency-selecting system.

If the speed of signaling is 30 words per minute and if $n_r = 3$, we have for the ideal receiver

$$\frac{P_{I\,C\,W.}}{P_s} = \frac{E^2p}{E_s^2}\,(0.008) \qquad (85)$$

At a signaling speed of 150 words per minute, the ratio of I.C.W. signal power to static power will be one-fifth as large as at 30 words per minute.

36. The Reception of Radio Telephone Signals through Stray Interference.

In the section on telephony it was shown that for distortionless reception the ideal receiving system must pass a band of frequencies running from $f_0 - f_c$ cycles per second to $f_0 + f_c$ cycles per second where f_c was about 5,000 cycles. Under these conditions the power picked up by the system from stray disturbances is given by Eq. (82) of Sec. 35. Since the ideal system picks up energy only from generators which have frequencies lying in the band which must be transmitted, it follows that the ideal system as described is the best possible frequency-selection system for receiving telephone messages through stray interference. Since f_c for telephony is $5{,}000 \div 30 = 167$ times as large as for I.C.W. reception at 30 words per minute, 167 times as much stray energy must be picked up in a telephone receiver as in an I.C.W. receiver.

37. The I.C.W. Transmitter and the Spark Transmitter as Sources of Interference.

It has been shown that the voltage induced in a receiving antenna by an I.C.W. transmitter may be replaced by a group of generators having the correct frequencies and correct voltages. Any I.C.W. transmitting station has generators with frequencies located in all frequency bands. It has also been shown that all receiving systems must transmit a band of frequencies centered upon the operating frequency of the station whose signals it is desired to receive. The receiving system must necessarily, therefore, pick up energy from all I.C.W. stations which induce a voltage in the receiving antenna. With a given receiving system the

energy picked up from an interferent I.C.W. station depends upon the difference between the operating frequency of the interferent station and the operating frequency of the correspondent station and the manner in which the voltage assigned to any generator which replaces the interferent voltage in the receiving antenna varies with the frequency assigned to it. The curve of Fig. 48 gives a graphical picture of the dependence of the voltage assigned to any generator upon the frequency assigned to the generator. From Eq. (59) we draw the conclusion that 90 per cent of the energy available in the waves is associated with a band of frequencies $\frac{3}{T}$ cycles wide and centered on the operating frequency. At 30 words per minute $\frac{3}{T}$ has a value of about 60 cycles and at 150 words per minute $\frac{3}{T}$ has a value of about 300 cycles. We thus come to the conclusions that the energy associated with the waves of an I.C.W. transmitter is confined to a narrow band of frequencies and the width of this band varies directly with the signal speed. The above statement holds true if the I.C.W. transmitter has no harmonics. If harmonics are present in the waves sent out by the transmitter, the voltages assigned to generators having frequencies in the vicinity of the harmonic will vary as shown by Fig. 48 for the fundamental frequency, and any receiving station having a transmitted band in the vicinity of one of the harmonics will pick up an appreciable amount of power from the transmitter.

The spark transmitter causes much more interference than the I.C.W. transmitter. This fact is brought out in a striking manner if the curves of Fig 50 are compared with those of Fig. 48. The voltage assigned to a generator falls off slowly as the frequency departs from the operating frequency in the case of the spark transmitter. It is evident from an examination of the curves of Fig. 50 and from the discussion of Sec. 32 that the interference caused by a spark station is dependent upon the logarithmic decrement

times the frequency rather than upon the logarithmic decrement. If $\alpha = \delta f_0 = 10^4$, then Eqs. (76) and (77) show that 82 per cent of the energy in the waves is associated with a band of frequencies 10,000 cycles wide centered on the operating frequency f_0. If $\alpha = \delta f_0 = 10^5$, then 82 per cent of the energy is associated with a band of frequencies approximately 100,000 cycles wide centered on the operating frequency. This is in striking contrast to the I.C.W. case where most of the energy is associated with a band of frequencies 60 to 300 cycles wide. From the above discussion it is evident that a station operating at a frequency of 10^6 cycles per second and having a logarithmic decrement of 0.01 has its energy spread over the same band width as a station operating at 10^5 cycles per second and having a logarithmic decrement equal to 0.1.

38. General Conclusions on the Extent to Which Interference Can Be Mitigated by Frequency Selecting Systems.

All sources of interference to radio reception and all sources of signals have definite frequency spectra, and for convenience we can replace the voltages induced in an antenna by a group of generators having the correct voltages and frequencies. In order to receive signals the receiving system must pass freely the currents due to all generators having frequencies in a given band. This band is centered on the operating frequency of the station from which it is desired to receive signals. The width of the band is determined by the class of signals which it is desired to receive. If the interferent voltages have generators with frequencies in this transmitted band, the frequency-selection system cannot eliminate the currents due to these generators. Since the ideal receiver as specified in this chapter utilizes the maximum possible power from generators lying within the band of frequencies which must be passed and utilizes no power from generators which have frequencies outside this band, it is evident that the interference obtained in the ideal receiver is the minimum which can be obtained with frequency-selecting systems. From this it is evident that the minimum interference which it is

possible to obtain depends upon the ratio of the voltage of the signal generators to the voltage of the interferent generators which lie in the transmitted band and upon the width of the band of frequencies which it is necessary to transmit. The width of the band of frequencies which must be transmitted depends upon the class of signals which are to be received. Thus the necessary band width for I.C.W. signals at 30 words per minute is only 60 cycles, and at 150 words per minute it is 300 cycles. The necessary transmitted band width for telephony is about 10,000 cycles, and the necessary transmitted band width for spark telegraphy varies from 10,000 to 100,000 cycles, depending upon the product of the logarithmic decrement times the operating frequency.

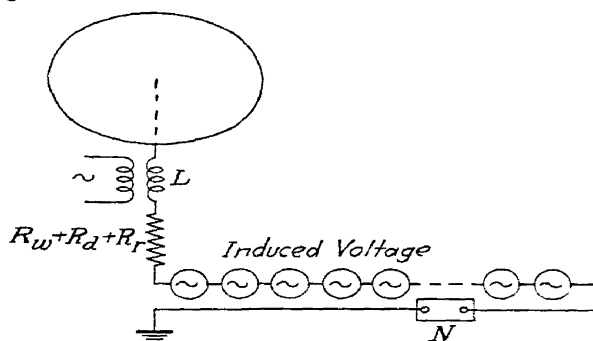



FIG. 52 —Simple series receiving system

39. Theory of the Simple Series Antenna Circuit Associated with a Pure Resistance Neutralizer.

In the preceding sections we have discussed the nature of the voltage induced in a receiving antenna by signals and by interference, and an ideal frequency-selection system for receiving signals through interference has been described. Let us now consider the simple series receiving circuit shown by Fig. 52. This circuit consists of an elevated capacity network A, a tuning inductance L, a detector which has a resistance R_d, and a resistance neutralizer which reduces the effective resistance of the system by the amount N. The circuit has a wasteful resistance R_w and a radiation resistance R_r. The induced voltage is represented by the

group of generators. It has been shown that the ideal receiver should act as a pure resistance of magnitude $2R_r$ to a continuous band of frequencies the width of which depends upon the kind of signals to be received and that it should eliminate the currents of all other frequencies. Of course it is impossible to adjust the simple series circuit so that it will act as a resistance to all generators whose frequencies lie in a given band and will eliminate the currents due to all other generators. We will therefore tune the circuit to the operating frequency f_0 and adjust the circuit so that it approximates the ideal circuit as closely as possible.

Let the frequency band which the ideal circuit must pass for any given kind of signals be denoted by B_w. Thus for the reception of I.C.W. signals at 30 words per minute, B_w is equal to about 60 cycles; for the reception of I.C.W. signals at 150 words per minute, B_w is about 300. For double side-band telephony, B_w is about 10,000. For the reception of spark telegraphy, B_w varies from 10,000 to 100,000. Now the impedance of the circuit to the generator whose frequency is f_0 is

$$Z_0 = R_n = R_r + R_d + R_w - N = \gamma R_t \tag{86}$$

The net reactance of the circuit to a generator whose frequency is $f_0 + a$ is

$$X_n = 2\pi L(f_0 + a) - \frac{1}{2\pi f_0 C\left(1 + \frac{a}{f_0}\right)}$$

But since

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

we have

$$X_n = 2\pi L(f_0 + a) - 2\pi f_0 L\left(1 + \frac{a}{f_0}\right)^{-1}$$

$$\left(1 + \frac{a}{f_0}\right)^{-1} = 1 - \frac{a}{f_0} \text{ very closely if } \frac{a}{f_0} < 0.05$$

Therefore we may write

$$X_n = 2(2\pi La) \tag{87}$$

Let us now assume that in order to have the circuit respond to as few frequencies as possible and still give satisfactory signal reception, the reactance of the circuit to the generator whose frequency is $f_0 \pm \frac{B_w}{2}$ must be βR_n. If we set a in Eq. (87) equal to $\frac{B_w}{2}$, we have

$$\left.\begin{aligned} \beta R_n &= 2\pi L B_w \\ \frac{2L}{R_n} &= T_c = \frac{\beta}{\pi B_w} \end{aligned}\right\} \qquad (88)$$

In Eq. (88), T_c is the time-constant of the circuit. In order to keep down interference, β should be large. If it is made too large, however, the reception of signals will not be satisfactory. Calculations based upon the shape of the antenna current wave in I.C.W. telegraphy show that when $n_r = 3$ ($B_w = 60$ at 30 words per minute), β cannot be assigned a value much larger than 2 if the dots and spaces are to be discernible. For I.C.W. and spark telegraphy we will therefore fix the upper value of β at 2. In telephone reception the value assigned to β determines the distortion. In the audio-frequency amplifier and in the loud speakers used in radio receiving sets there is considerable distortion, and these systems generally discriminate against the low frequencies in favor of the upper tones. A value of 2 for β in telephony should not be excessive. This point will be discussed further when we take up the power relations in telephony. If the upper value of β is taken equal to 2, then the maximum values which the time constant of the simple series receiving circuit may have for various classes of signals are found from Eq. (88) to be as follows: I.C.W. at 30 words per minute,

$$T_c \text{ (maximum value)} = \frac{2}{\pi 60} = 0.0106 \qquad (89)$$

I.C.W. at 150 words per minute,

$$T_c \text{ (maximum value)} = \frac{2}{\pi 300} = 0.0021 \qquad (90)$$

Spark telegraphy $\alpha = \delta f_0 = 10^4$

$$T_c \text{ (maximum value)} = \frac{2}{(\pi)10{,}000} = 6.35 \times 10^{-5} \quad (91)$$

Spark telegraphy $\alpha = \delta f_0 = 10^5$

$$T_c \text{ (maximum value)} = \frac{1}{(\pi)100{,}000} = 6.35 \times 10^{-6} \quad (92)$$

Telephony, double side-band transmission

$$T_c \text{ (maximum value)} = \frac{2}{(\pi)10{,}000} = 6.35 \times 10^{-5} \quad (93)$$

In Sec. 27 it was shown that for maximum power delivery to the detector, the detector resistance should be

$$R_d = R_r + R_w \quad (94)$$

In a communication system it is of more importance to satisfy Eq. (88) than Eq. (94) because Eq. (88) fixes the selectivity of the simple series circuit at the maximum possible value. Since $R_n = R_r + R_w + R_d - N$, it may not always be possible to satisfy Eq. (94), so we write

$$R_d = \rho(R_r + R_w) \quad (95)$$

If we are dealing with an antenna circuit having a fixed radiation and a fixed wasteful resistance associated with a given resistance neutralizer, the value of ρ is fixed by satisfying Eq. (88). Section 27 discusses the question of designing the antenna so as to be able to assign the best possible value, namely, unity, to ρ. This section should be reviewed at this point.

In order to show the effect of the time-constant upon the current in a circuit consisting of inductance, resistance, and capacitance in series when excited by a voltage of the kind which we have assumed to be induced in a receiving antenna by I.C.W. waves, the oscillograms given by Figs. 52*a* and 52*b* are presented. The circuits used in taking these oscillograms are given by the figures. The voltage impressed in the circuit for each case is proportional to the integral of the curve marked I_1. The voltage has the

general characteristics assumed for the I.C.W. case. The curves marked I_2 give the current in the tuned circuit. The combined time duration of a dot and a space is $\frac{1}{30}$ second. This corresponds to a signal speed of about 90 words per minute. For the case shown by Fig. 52*a*,

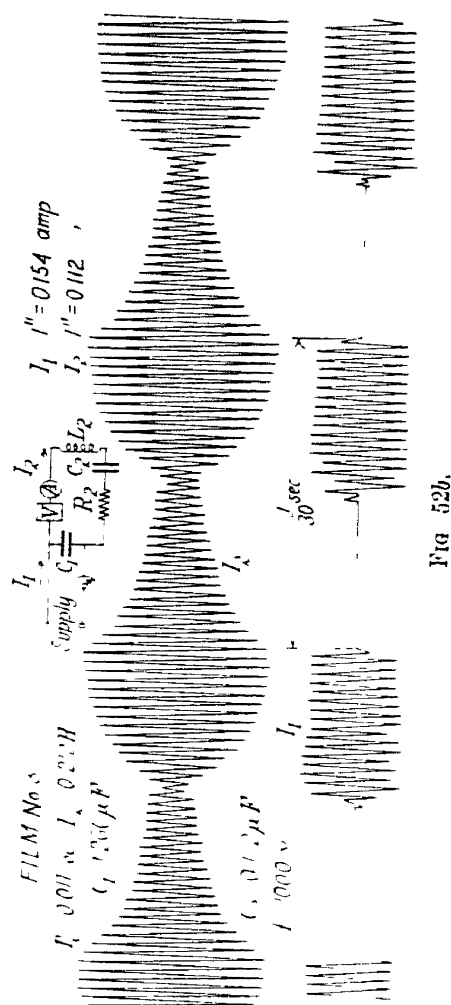
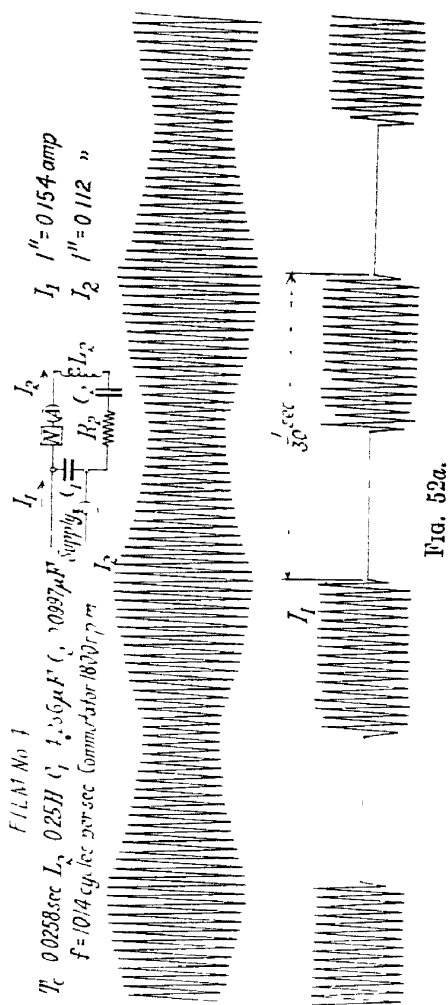



Fig. 52*a*.

Fig. 52*b*.

the time-constant is 0.026 second. The ratio of the maximum current peak to the minimum current peak is too small for a distinguishable signal. For the case shown by Fig. 52*b*, the time-constant is 0.011 second. The ratio of the maximum to the minimum current peak is greater

than for the previous case but is still not great enough to permit the distinguishing of the signals by audible methods, although the signals might be discerned by visual methods.

40. Power Relations in the Simple Series Circuit Receiving I.C.W. Signals.

If we refer again to the curve of Fig. 48 we see that, in the I.C.W. case, generators are located on a frequency scale every $f_0 + \frac{N}{2T}$ cycles where $n = 0, \pm 1, \pm 3, \pm 5$, etc. The reactance of the circuit to the nth generator as given by Eq. (87) then is

$$X_n = 2\pi L \frac{n}{T} \tag{96}$$

If in Eq. (88) we substitute

$$B_w = \frac{N_\tau}{T} \tag{97}$$

and solve for L and put this value of L in Eq. (96), we find that the reactance of the circuit to the nth generator is

$$X_n = \beta R_n \frac{n}{n_\tau} \tag{98}$$

If the r.m.s. voltage of the nth generator is represented by E_n, then the average power delivered to the detector by the nth generator is

$$P_n = \frac{E_n^2 R_d}{R_n^2\left[1 + \beta^2\left(\frac{n}{n_\tau}\right)^2\right]} \tag{99}$$

From Eqs. (45) and (48) of Sec. 31, we write

$$E_n^2 = \frac{E^2}{8} \text{ for } n = 0 \tag{100}$$

$$= \frac{E^2}{2n^2\pi^2} \text{ for all other values of } n \tag{101}$$

The total average power supplied to the detector then is

$$P = \frac{E^2 R_d}{2R_n^2}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_n \frac{1}{n^2\left[1 + \beta^2\left(\frac{n}{n_\tau}\right)^2\right]}\right] \tag{102}$$

$$n = 1, 3, 5, \text{ etc.}$$

The factor 2 multiplies the summation term of Eq. (102) because there are two generators having the voltage given by Eq. (101). The summation term in Eq. (102) converges rapidly because of the presence of n^4 in the denominator. If, as in Sec. 27 we write

$$k = \frac{R_r + R_w}{R_r} \tag{103}$$

and make use of Eq. (95), we obtain

$$R_n = \gamma R_t = \gamma(R_r + R_w + R_d) = \gamma k R_r(1 + \rho) \tag{104}$$

$$R_d = \rho(R_r + R_w) = \rho k R_r \tag{105}$$

Upon substituting these values in Eq. (102) there results

$$P = \frac{\rho E^2}{\gamma^2 k R_r(1 + \rho)^2}\left[\frac{1}{8} + \frac{1}{\pi^2}\sum_n \frac{1}{n^2\left[1 + \beta^2\frac{n^2}{n_T^2}\right]}\right] \tag{106}$$

$$n = 1,\ 3,\ 5,\ \text{etc.}$$

γ is the reduction factor of the resistance neutralizer. If the neutralizer were removed from the circuit and no other changes were made, the net resistance would be R_t and the reactance to each generator would remain at the value

$$X_n = R_n\beta\frac{n}{n_T} = \gamma R_t\beta\frac{n}{n_T} \tag{107}$$

The power delivered to the detector would be

$$P_{un} = \frac{\rho E^2}{k R_r(1 + \rho)^2}\left[\frac{1}{8} + \frac{1}{\pi^2}\sum_n \frac{1}{n^2\left(1 + \gamma^2\beta^2\frac{n^2}{n_T^2}\right)}\right] \tag{108}$$

The regenerative amplification therefore is

$$\frac{P}{P_{wn}} = \frac{\frac{1}{8} + \frac{1}{\pi^2}\sum_n \frac{1}{n^2\left(1 + \beta^2\frac{n^2}{n_T^2}\right)}}{\gamma^2\left[\frac{1}{8} + \frac{1}{\pi^2}\sum_n \frac{1}{n^2\left(1 + \gamma^2\beta^2\frac{n^2}{n_T^2}\right)}\right]} \tag{109}$$

$$n = 1,\ 3,\ 5,\ \text{etc.}$$

If $n_r = 3$, $\beta = 2$, and $\gamma = 0.01$, the regenerative amplification is

$$\text{regenerative amplification} = \frac{10^4[0.125 + 0.07 + 0.00225 + 0.00035 + \cdots]}{0.125 + 0.1012 + 0.0112 + 0.004 + \cdots}$$

$$= 10^4\frac{0.197}{0.241} = 0.82 \times 10^4 \qquad (110)$$

If the calculation of regenerative amplification were made on the basis of the steady-state theory, we would arrive at the value 10^4. The regenerative amplification of an I.C.W. signal in a simple series circuit having the best possible selectivity is therefore 82 per cent of the regenerative amplification at resonance calculated on the basis of steady-state theory.

To arrive at the energy delivered to the circuit by strays, we make use of the information obtained in Secs. 34 and 35. It was pointed out in these sections that the voltages assigned to the generators representing the stray voltage which have frequencies lying in a narrow band in the radio range will be about the same for all frequencies. Further, because of the random nature of static, one frequency is as likely to be a generator frequency as any other frequency. In the discussion of Eq. (84) of Sec. 35 it was pointed out that $\frac{E_s^2}{p}$ characterized the energy level of the strays in the vicinity of the operating frequency of the I.C.W. communication system. That is, the mean-squared current which would flow in the ideal circuit having a transmitted band width B_w cycles wide and a net effective resistance equal to R_n would be

$$I_s^2 = \frac{E_s^2}{2pR_n^2}B_w \qquad (111)$$

Because of the random nature of a stray we may take this current as being uniformly distributed over all frequencies in the band, and for convenience we will write

$$W_s = \frac{E_s^2}{2p} \qquad (112)$$

The mean-square current in the ideal circuit due to strays then will be

$$I_s^2 = \frac{W_s B_w}{R_n^2} \tag{113}$$

If the circuit has reactance as well as resistance, the mean-square current due to generators having frequencies in an elementary band df will be

$$dI_s^2 = \frac{W_s}{Z^2_{(f)}} df \tag{114}$$

The term W_s will be called the energy level of the strays in the vicinity of the operating frequency. The value of W_s might be estimated from the work of Sec. 34. The work of Sec. 34 and other observations indicate that its value falls off as the operating frequency of the communication system is increased. The symbolism $Z_{(f)}$ indicates that Z is a function of the frequency. If we solve Eq. (88) for L and put this value of L in Eq. (87), we find that the net reactance of the simple series circuit to a frequency a cycles different from the resonant frequency of the circuit is

$$X_n = \frac{2\beta R_n}{B_w} a = \frac{2\beta\gamma R_t}{B_w} a \tag{115}$$

The mean-square current in the circuit due to strays is found from Eqs. (114) and (115) to be

$$I_s^2 = \frac{W_s}{\gamma^2 R_t^2} \int_{-\infty}^{+\infty} \frac{da}{\left(1 + \frac{4\beta^2}{B_w^2} a^2\right)} = \frac{W_s B_w}{\gamma^2 R_t^2 2\beta} \tan^{-1}\left(\frac{2\beta}{B_w} a\right) \Bigg|_{-\infty}^{+\infty}$$

$$= \frac{W_s B_w \pi}{2\beta\gamma^2 R_t^2} \tag{116}$$

The justification for summing from $-\infty$ to $+\infty$ lies in the fact that by far the greatest contribution to the integral is made by frequencies in the vicinity of f_0 because of the

shape of the resonance curve. The power supplied to the detector by strays then is

$$P_s = I_s^2 R_d = \frac{W_s B_w \pi R_d}{2\beta\gamma^2 R_t^2} = \frac{W_s B_w \rho \pi}{2\beta k \gamma^2 R_r (1+\rho)^2} \qquad (117)$$

The ratio of signal power to stray power is

$$\frac{P_{I.C.W.}}{P_s} = \frac{2\beta E^2}{W_s B_w \pi}\left[\frac{1}{8} + \frac{1}{\pi^2}\sum_n \frac{1}{n^2\left(1 + \beta^2 \frac{n^2}{n_r^2}\right)}\right] \qquad (118)$$

$$n = 1, 3, 5 \text{ etc.}$$

By the aid of Eqs. (88) and (97), Eq. (118) can be written in terms of the time constant of the circuit as follows:

$$\frac{P_{I\,C\,W}}{P_s} = \frac{2E^2 T_c}{W_s}\left[\frac{1}{8} + \frac{1}{\pi^2}\sum_n \frac{1}{n^2\left(1 + \frac{n^2\pi^2 T_c^2}{T^2}\right)}\right] \qquad (119)$$

The effect of the time-constant on the summation term of Eq. (119) is relatively small. If T_c were given the value 0, the second term in the brackets would be equal to 0.125. If T_c were assigned the value ∞, the second term in the brackets would be zero. It follows therefore that the signal-stray power ratio will be a maximum when T_c is assigned the largest possible value. That is, the signal-stray power ratio varies almost directly with β and inversely with B_w. Now there is an upper limit to the value which can be assigned to β if the signals are to be discernible and B_w is fixed by the speed of signaling. Therefore there is an upper limit to the signal-stray power ratio which can be obtained with the simple series circuit, and this value is fixed by the speed of signal transmission. If a large amount of copper were used in the coil and if great precautions were taken to keep down the wasteful resistance of the system, this maximum selectivity might possibly be obtained without the use of a resistance neutralizer. It is far cheaper and easier, however, to obtain the time-constant necessary to give maximum selectivity by associating a triode with the circuit in such a way as to lower its effective resistance.

From Eq. (84) of Sec. 35 and Eq. (118) of this section, the ratio of the I.C.W. signal power to the stray power for the simple series circuit divided by the same ratio for the ideal circuit is

$$\frac{P_{I\,C\,W.} \div P_s \text{ (series circuit)}}{P_{I\,C\,W} \div P_s \text{ (ideal circuit)}} = \frac{\beta}{\pi} \frac{\left[\frac{1}{8} + \frac{1}{\pi^2}\sum_{n'} \frac{1}{n^2\left(1 + \beta^2\frac{n^2}{n_r^2}\right)}\right]}{\left[\frac{1}{8} + \frac{1}{\pi^2}\sum_{s}\frac{1}{s^2}\right]} \quad (120)$$

$$n = 1, 3, 5 \text{ etc.} \quad s = 1, 3, \ldots n_r$$

If $n_r = 3$ and $\beta = 2$, this ratio has the value 0.53. That is, it is possible to design the simple series receiving circuit so that its I.C.W. signal-stray power ratio is 53 per cent as great as this same ratio for the ideal circuit.

41. Power Relations in the Simple Series Circuit Receiving Telephone Signals.

Let the generators which replace the voltage induced in the receiving antenna have frequencies p cycles apart and let the generator whose frequency differs from the carrier frequency by a cycles have a peak voltage of $E_{t^{a}}$ volts. The reactance of the circuit to the generators whose frequencies are $f_0 + \frac{1}{2}B_w$ according to Eq. (88) of Sec. 34 is $\beta R_n = \beta\gamma R_t$. The power delivered to the detector by each of these generators then is

$$P_{c1} = \frac{E^2R_d}{2R_n^2(1 + \beta^2)}$$

For distortionless reception this power should be

$$P_{c2} = \frac{E^2R_d}{2R_n^2}$$

The ratio of these two powers gives a measure of the distortion of the circuit. This ratio is

$$D = \frac{P_{c1}}{P_{c2}} = \frac{1}{1 + \beta^2} \quad (121)$$

If $\beta = 2$, the highest important voice or musical note will supply one-fifth as much power to the circuit as it should supply for distortionless reception. This amount of distortion will probably not be excessive. If we take all frequencies in the band as the location of generator frequencies and if we represent the telephone energy level by

$$W_{t(a)} = \frac{E^2_{t(a)}}{2p} \tag{122}$$

then by analogy with Eqs. (111) to (116) of Sec. 40, we write

$$I_t^2 = \frac{1}{\gamma^2 R_{\cdot t}^2} \int_{-\frac{1}{2}B_w}^{+\frac{1}{2}B_w} \frac{W_{t(a)}}{\left(1 + \frac{4\beta^2}{B_w^2}a^2\right)} da \tag{123}$$

In Eq. (123), I_t^2 is the mean-square value of the antenna current due to the telephone generators. The summation limits are taken as they are because $W_{t(a)}$ is assumed equal to zero outside the band of frequencies $f_0 \pm \frac{1}{2}B_w$. The average power delivered to the detector by the telephone signals is

$$P_t = I_t^2 R_d \tag{124}$$

If the neutralizer is removed from the circuit and no other change is made, the mean-square current becomes

$$I_t^2 \text{ (without neutralizer)} = \frac{1}{R_t^2} \int_{-\frac{1}{2}B_w}^{+\frac{1}{2}B_w} \frac{W_{t(a)}}{\left(1 + \frac{4\gamma^2\beta^2}{B_w^2}a^2\right)} da \tag{125}$$

The regenerative amplification then is

$$\text{regenerative amplification} = \frac{I_t^2 \text{ (with neutralizer)}}{I_t^2 \text{ (without neutralizer)}} \tag{126}$$

If $W_{t(a)}$ is taken as a constant, then by analogy with Eq. (116) of Sec. 40, we write

$$I_t^2 = \frac{W_t B_w}{\gamma^2 R_t^2 2\beta} \tan^{-1}\left(\frac{2\beta}{B_w}a\right)\Bigg|_{-\frac{1}{2}B_w}^{+\frac{1}{2}B_w} = \frac{W_t B_w}{\gamma^2 R_t^2 \beta} \tan^{-1}\beta \tag{127}$$

$$I_t^2 \text{ (without neutralizer)} = \frac{W_t B_w}{\gamma R_t^2 \beta} \tan^{-1} (\gamma \beta) \qquad (128)$$

$$\text{regenerative amplification} = \frac{1}{\gamma} \frac{\tan^{-1} \beta}{\tan^{-1} (\gamma \beta)} \qquad (129)$$

If $\gamma = 0.01$, and $\beta = 2$, we have

$$\text{regenerative amplification} = 100 \frac{\tan^{-1} 2}{\tan^{-1} (0.02)} =$$

$$100 \frac{63.43}{1.15} = 5{,}510 \qquad (130)$$

The stray power delivered to the detector is given by Eq. (117) of Sec. 40. If the telephone energy level is taken constant over the band width B_w, the ratio of signal power to stray power is

$$\frac{P_t}{P_s} = \frac{W_t}{W_s} \frac{2}{\pi} \tan^{-1} \beta \qquad (131)$$

If $\beta = 2$, $\tan^{-1} \beta = (0.703)\frac{\pi}{2}$ and

$$\frac{P_t}{P_s} = \frac{W_t}{W_s} (0.703) \qquad (132)$$

If the stray energy level in the vicinity of the carrier frequency is constant over the band width B_w and if the telephone energy level is also constant over the band width B_w, then the ratio of the telephone signal power to the stray power for the ideal circuit is

$$\left(\frac{P_t}{P_s}\right)_{\text{ideal circuit}} = \frac{W_t}{W_s} \qquad (133)$$

From this we see that the simple series circuit which cuts the power received from the highest important voice or musical frequency to one-fifth the value which it should have for distortionless reception has a signal-stray power ratio 70 per cent as large as the signal-stray power ratio of the ideal receiver.

CHAPTER VI

TRIODE CIRCUIT EQUATIONS

In the preceding chapters, the equations for a number of triode circuits have been developed and discussed. This chapter will be devoted to the development and discussion of the equations of other important triode circuits. The equations for the alternating potentials and currents only will be written down, but it should be kept in mind that superposed on these alternating quantities there is a system of continuous currents and potentials.

42. Triode with a Tuned Circuit in the Plate Branch. This circuit is shown in Fig. 53. In deriving the equations for this circuit, we shall assume that R_1 includes the resistance of the coil in the plate circuit. This greatly simplifies the equations, and the results are not greatly in error if, as is generally the case, the circulating current I_1 is large compared to the plate current I_p. G_g and G_{cg} are assumed to equal zero.

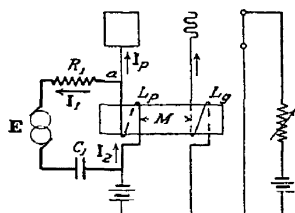



FIG 53 —Triode with a tuned circuit in the plate branch.

Upon summing currents to zero at a there results

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_p \tag{1}$$

Upon applying Kirchoff's e.m.f. law to the generator circuit there results

$$\mathbf{E} - \mathbf{R}_1\mathbf{I}_1 + jX_c\mathbf{I}_1 - jX_p(\mathbf{I}_1 + \mathbf{I}_p) = 0 \tag{2}$$

The plate voltage is

$$\mathbf{E}_p = -jX_p(\mathbf{I}_1 + \mathbf{I}_p) \tag{3}$$

The grid voltage is

$$\mathbf{E}_g = -jX_m(\mathbf{I}_1 + \mathbf{I}_p) \tag{4}$$

The plate space-current then is

$$\mathbf{I}_p = \mathbf{E}_g G_{cp} + \mathbf{E}_p G_p = -jX_m(\mathbf{I}_1 + \mathbf{I}_p)G_{cp} - jX_p(\mathbf{I}_1 + \mathbf{I}_p)G_p \quad (5)$$

Solving Eq. (5) for $\mathbf{I}_p$, there results

$$\mathbf{I}_p = -\frac{j(X_m G_{cp} + X_p G_p)}{1 + j(X_m G_{cp} + X_p G_p)}\mathbf{I}_1 \quad (6)$$

Rationalizing Eq. (6),

$$\mathbf{I}_p = -\frac{j(X_m G_{cp} + X_p G_p) + (X_m G_{cp} + X_p G_p)^2}{1 + (X_m G_{cp} + X_p G_p)^2}\mathbf{I}_1 \quad (7)$$

Substituting Eq. (7) in Eq. (2), we obtain

$$\mathbf{E} - \mathbf{I}_1\left\{R_1 + \frac{X_p(X_m G_{cp} + X_p G_p)}{1 + (X_m G_{cp} + X_p G_p)^2} + j\left[X_p - X_c - \frac{X_p(X_m G_{cp} + X_p G_p)^2}{1 + (X_m G_{cp} + X_p G_p)^2}\right]\right\} = 0 \quad (8)$$

Equation (8) is in the standard form, namely,

$$\mathbf{E} - \mathbf{I}_1\left[R_1 - N + j\,(X_n + X_A)\right] = 0 \quad (9)$$

Thus we write

$$N = -\frac{X_p(X_m G_{cp} + X_p G_p)}{1 + (X_m G_{cp} + X_p G_p)^2} \quad (10)$$

$$X_A = -\frac{X_p(X_m G_{cp} + X_p G_p)^2}{1 + (X_m G_{cp} + X_p G_p)^2} \quad (11)$$

If N is to be positive, that is, if the triode is to lower the effective resistance of the generator circuit, X_m must be negative and $X_m G_{cp}$ must be greater in absolute value than $X_p G_p$. X_m is negative when the plate and grid coils are wound in opposite directions, as shown by Fig. 53.

If the circulating current is large compared to the plate space current, Eqs. (3) and (4) reduce very closely to

$$\mathbf{E}_p = -jX_p\mathbf{I}_1 \text{ (approximately)} \quad (12)$$

$$\mathbf{E}_g = -jX_m\mathbf{I}_1 \text{ (approximately)} \quad (13)$$

From Eqs. (12) and (13) we see that if E_p and E_g are to be 180 degrees out of phase, X_m must be negative. (X_p is always positive.)

$$\frac{E_p}{E_g} = \frac{X_p}{|X_m|} \text{ (approximately)} \tag{14}$$

From Eqs. (12) and (14), we write

$$U = X_p = \omega L_p \text{ (approximately)} \tag{15}$$

$$\delta = \frac{X_p}{|X_m|} = \frac{\omega L_p}{|\omega M|} = \frac{L_p}{|M|} \text{ (approximately)} \tag{16}$$

The approximate expression for N when X_m is negative then is

$$N = U^2\left(\frac{G_{cp}}{\delta} - G_p\right) \text{ (approximately)}$$

$$= X_p^2\left(\frac{G_{cp}|X_m|}{X_p} - G_p\right) \text{ (approximately)}$$

$$N = X_p(|X_m|G_{cp} - X_pG_p) \text{ (approximately)} \tag{17}$$

The approximate vector diagram for this circuit is shown in Fig. 54. The circulating current $\mathbf{I}_1$ is taken as the

$\mathbf{E}_g = -jX_m\mathbf{I}_1$

$\mathbf{E}_g G_{cp}$

$\mathbf{I}_p = \mathbf{E}_g G_{cp} + \mathbf{E}_p G_p$

$\mathbf{I}_1$

$\mathbf{E}_p G_p$

$\mathbf{E}_p = -jX_p\mathbf{I}_1$

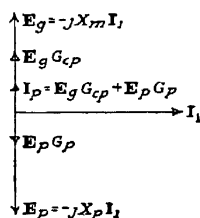



Fig. 54.—Approximate vector diagram for the circuit of Fig. 53 when X_m is negative.

reference vector. If X_m is negative, the grid voltage leads this current by 90 degrees. The plate voltage lags $\mathbf{I}_1$ by 90 degrees, thus making the plate and grid voltages 180 degrees out of phase. The plate space current is drawn for the condition that E_gG_{cp} is greater than E_pG_p. The total plate current then is 180 degrees out of phase with the plate voltage, and conditions are correct for the triode to function as a generator of alternating power.

43. Triode with a Tuned Circuit in the Grid Branch.

The diagram for this circuit is shown in Fig. 55. In the equations which follow, the following abbreviations are used:

$$X_c \text{ for } \frac{1}{\omega C_1}$$

X_g for ωL_g $\qquad$ X_m for ωM

X_p for ωL_p $\qquad$ D for $1 + R_pG_p$

Upon summing the voltages around the tuned circuit in the grid branch, there results

$$\mathbf{E} - R_1\mathbf{I}_1 - jX_g\mathbf{I}_1 + jX_c\mathbf{I}_1 - jX_m\mathbf{I}_p = 0 \qquad (18)$$

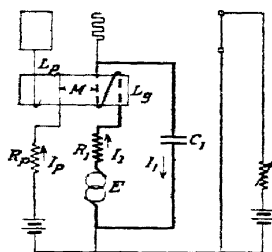



FIG. 55.—Triode with a tuned circuit in the grid branch.

The grid voltage is the voltage of the condenser in a direction opposite to the arrow direction. The voltage of the condenser in the arrow direction is $+jX_c\mathbf{I}_1$. Therefore the grid voltage is

$$\mathbf{E}_g = -jX_c\mathbf{I}_1 \qquad (19)$$

The plate voltage is

$$\mathbf{E}_p = -jX_m\mathbf{I}_1 - jX_p\mathbf{I}_p - R_p\mathbf{I}_p \qquad (20)$$

The plate space current then is

$$\mathbf{I}_p = -jX_c\mathbf{I}_1G_{cp} - jX_m\mathbf{I}_1G_p - jX_p\mathbf{I}_pG_p - R_p\mathbf{I}_pG_p \qquad (21)$$

Solving Eq. (21) for $\mathbf{I}_p$ gives

$$\mathbf{I}_p = -\frac{j(X_cG_{cp} + X_mG_p)}{1 + R_pG_p + jX_pG_p}\mathbf{I}_1 \qquad (22)$$

Upon rationalizing Eq. (22), we obtain

$$\mathbf{I}_p = -\frac{j(X_cG_{cp} + X_mG_p)D + (X_cG_{cp} + X_mG_p)X_pG_p}{D^2 + X_p^2G_p^2}\mathbf{I}_1 \qquad (23)$$

Substituting $\mathbf{I}_p$ as given by Eq. (23) in Eq. (18), we obtain:

$$\mathbf{E} - \mathbf{I}_1\left\{R_1 + \frac{X_m(X_cG_{cp} + X_mG_p)D}{D^2 + X_p^2G_p^2} + j\left[X_g - X_c - \frac{X_m(X_cG_{cp} + X_mG_p)X_pG_p}{D^2 + X_p^2G_p^2}\right]\right\} = 0 \quad (24)$$

Equation (24) is in the standard form given by Eq. (9), and we therefore write

$$N = -\frac{X_m(X_cG_{cp} + X_mG_p)D}{D^2 + X_p^2G_p^2} \quad (25)$$

Now all terms in Eq. (25) are necessarily positive except X_m which may be either positive or negative. If N is to be positive, that is, if the resistance is to be lowered by the triode, X_m must be negative and

$$X_cG_{cp} \text{ must be greater than } |X_mG_p| \quad (26)$$

If the voltage induced in the plate circuit by the current I_1 is large compared to the plate voltage due to I_p, then Eq. (20) becomes

$$\mathbf{E}_p = -jX_m\mathbf{I}_1 \quad (27)$$

The approximate value of U for this circuit then is

$$U = |X_m| = \omega|M| \quad (28)$$

$$\frac{E_p}{E_g} = \delta = \frac{|X_m|}{X_c}$$

$\mathbf{E}_p = -jX_m\mathbf{I}_1$, $\mathbf{E}_pG_p$, $\mathbf{I}_1$, $\mathbf{I}_p$, $\mathbf{E}_gG_{cp}$, $\mathbf{E}_g = -jX_c\mathbf{I}_1$

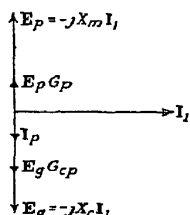



FIG 56 —Approximate vector diagram for the circuit of Fig 55 when X_m is negative

If X_m is negative, the approximate expression for N is

$$N = U^2\left(\frac{G_{cp}}{\delta} - G_p\right) = X_m^2\left(\frac{G_{cp}X_c}{|X|_m} - G_p\right)$$
$$= |X_m|(G_{cp}X_c - |X_m|G_p) \quad (29)$$

The approximate vector diagram when M is negative and Eq. (26) is satisfied is given in Fig. 56.

44. Triode with a Tuned Circuit in Both the Plate and the Grid Branches.

The diagram for the circuit under consideration in this section is given in Fig. 57. The equations are written for the case in which the generator is located in the branch containing L_g. The equations are approximately correct for the case in which the generator is located in the branch containing C_g provided that the generator e.m.f. is small compared to the potential across the condenser C_g. This latter condition will be fulfilled by a system having small damping when the system is nearly resonant to the generator frequency.

Fig. 57 —Triode with a tuned circuit in both the plate and grid branches.

We shall first consider the approximate vector diagrams for the circuit. A number of cases arise. The coupling may be either positive or negative, and conditions also change as the generator frequency passes through the natural frequency of the tuned circuit in the plate branch.

Case I.—*M* is positive.

Subcase a.—The generator frequency is lower than the natural frequency of the tuned circuit in the plate branch. The vector diagram for this condition is shown in Fig. 58. The current $\mathbf{I}_1$ in the generator circuit is taken as the reference vector. The grid potential is the e.m.f. of the condenser C_g taken in a direction opposite to the arrow direction. The grid potential is equal to $-jX_{cg}\mathbf{I}_1$. This potential lags behind $\mathbf{I}_1$ by 90 degrees. The e.m.f. impressed in the tuned circuit in the plate branch is the voltage induced in the plate coil by the current $\mathbf{I}_1$. It is equal to $-jX_m\mathbf{I}_1$. Since X_m is a positive quantity, this voltage also lags behind $\mathbf{I}_1$ by 90 degrees. If I_p is small compared to I_2, then

$$\mathbf{I}_2 = \frac{-jX_m\mathbf{I}_1}{\mathbf{Z}_p} = \frac{-jX_m\mathbf{I}_1}{R_p + j(X_{Lp} - X_{cp})} \text{ (approximately)} \quad (30)$$

Since the generator frequency is lower than the natural frequency of the plate circuit, X_{cp} is larger than X_{Lp}, and $\mathbf{I}_2$ leads the voltage $-jX_m\mathbf{I}_1$ as shown. The plate potential

is the potential of the condenser C_p in a direction opposite to the arrow direction, that is it is approximately equal to $-jX_{cp}\mathbf{I}_2$. The plate potential thus lags behind $\mathbf{I}_2$ by 90 degrees as shown. Since $\mathbf{E}_p$ and $\mathbf{E}_g$ are less than 90 degrees out of phase, the triode absorbs power, that is, N is negative.

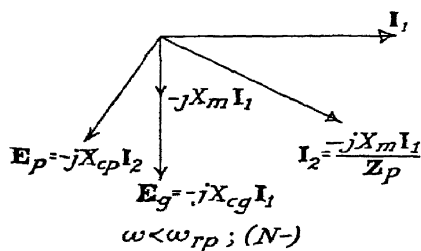



Fig. 58

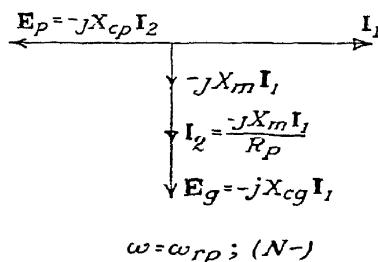



Fig 59.

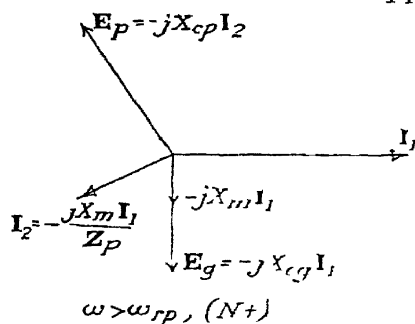



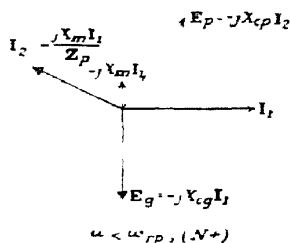
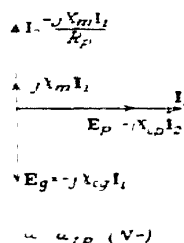
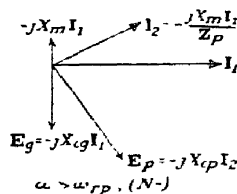
Fig. 60

Figs. 58, 59 and 60—Approximate vector diagrams for the circuit of Fig. 57 Cases for which M is positive.

Subcase b.—The generator frequency is equal to the natural frequency of the tuned circuit in the plate branch. The vector diagram for this case is given in Fig. 59. The net reactance of the tuned circuit in the plate branch is equal to zero; so $\mathbf{I}_2$ is in phase with the voltage vector $-jX_m\mathbf{I}_1$. The plate and grid potentials are displaced by substantially 90 degrees. No resistance neutralization takes place.

Subcase c.—The generator frequency is higher than the natural frequency of the tuned circuit in the plate branch. The vector diagram for this case is given in Fig. 60. The tuned circuit in the plate branch has a net inductive

reactance. The current $\mathbf{I}_2$ lags behind the voltage $-jX_m\mathbf{I}_1$. The plate and grid potentials are displaced by more than 90 degrees; so resistance neutralization may take place.

Case II.—M is negative. The same three subcases arise here as for case I. Since M is negative, the voltage $-jX_m\mathbf{I}_1$ leads $\mathbf{I}_1$ by 90 degrees. Therefore the vectors $\mathbf{I}_2$ and $\mathbf{E}_p$ which depend upon this voltage are rotated through 180 degrees in each subcase. Figure 61 shows the approximate vector diagram when the generator frequency is lower than the natural frequency of the plate tuned

FIGS 61, 62 and 63 —Approximate vector diagrams for the circuit of Fig 57. Cases for which M is negative.

circuit ($\omega < \omega_{rp}$). The plate and grid voltages are displaced by more than 90 degrees; so resistance neutralization may take place. Figure 62 shows the approximate vector diagram when the generator frequency equals the frequency of the plate tuned circuit. Since the plate and grid voltages are separated by substantially 90 degrees, no resistance neutralization can take place. Figure 63 shows that power absorption takes place when the generator frequency is higher than the natural frequency of the plate tuned circuit.

It is interesting to note the frequency of the sustained oscillations which might take place in the system of Fig. 57. Since there are two tuned coupled circuits, the system has two natural frequencies of oscillation, one above and one below the natural frequency of the plate circuit. If M is positive, resistance neutralization takes place only at frequencies above the natural frequency of the plate circuit. Therefore, the sustained oscillations will have a frequency equal to the higher of the two natural frequencies of the system. When M is positive, no sustained oscillations can take place at the lower of the two natural frequencies, as the triode absorbs power at this frequency. When M is negative, just the reverse is true. That is, the oscillations must take place at the lower of the two natural frequencies.

The approximate equations for the system will now be worked out. In these equations the following abbreviations are used:

X_{Lp} for ωL_p	X_{Lg} for ωL_g
X_{cp} for $\dfrac{1}{\omega C_p}$	X_{cg} for $\dfrac{1}{\omega C_g}$
X_p for $X_{Lp} - X_{cp}$	X_g for $X_{Lg} - X_{cg}$
Z_p for $R_p + jX_p$	X_m for ωM

Upon summing voltages around the grid tuned circuit, we obtain

$$\mathbf{E} - R_g\mathbf{I}_1 - jX_g\mathbf{I}_1 - jX_m\mathbf{I}_2 = 0 \tag{31}$$

Summing voltages around the plate circuit gives

$$-jX_m\mathbf{I}_1 - jX_{Lp}\mathbf{I}_2 + jX_{cp}(\mathbf{I}_2 - \mathbf{I}_p) - R_p\mathbf{I}_2 = 0 \tag{32}$$

The grid alternating voltage is

$$\mathbf{E}_g = -jX_{cg}\mathbf{I}_1 \tag{33}$$

If $\mathbf{I}_2$ is large compared to $\mathbf{I}_p$, the plate alternating voltage will be given very closely by

$$\mathbf{E}_p = -jX_{cp}\mathbf{I}_2 \tag{34}$$

The alternating plate space current will then be

$$\mathbf{I}_p = -j[X_{cg}\mathbf{I}_1G_{cp} + X_{cp}\mathbf{I}_2G_p] \tag{35}$$

Put Eq. (35) in Eq. (32) and obtain

$$jX_m\mathbf{I}_1 + \mathbf{Z}_p\mathbf{I}_2 + X_{cp}[X_{cg}\mathbf{I}_1G_{cp} + X_{cp}\mathbf{I}_2G_p] = 0 \quad (36)$$

Solving Eq. (36) for I_2 gives

$$\mathbf{I}_2 = -\frac{(jX_m + X_{cp}X_{cg}G_{cp})\mathbf{I}_1}{\mathbf{Z}_p + X_{cp}^2G_p} \quad (37)$$

Put Eq. (37) in Eq. (31) and obtain

$$\mathbf{E} - \mathbf{I}_1\left\{R_g + jX_g + jX_m\left[-\frac{jX_m + X_{cp}X_{cg}G_{cp}}{R_p + X_{cp}^2G_p + jX_p}\right]\right\} = 0 \quad (38)$$

Upon rationalizing the fraction of Eq. (38) and collecting real and j terms, there results

$$\mathbf{E} - \mathbf{I}_1\left\{R_g - \frac{X_mX_{cp}X_{cg}X_pG_{cp} - X_m^2X_{cp}^2G_p - X_m^2R_p}{(R_p + X_{cp}^2G_p)^2 + X_p^2} + j\left[X_g - \frac{X_m^2X_p + X_mX_{cp}^3X_{cg}G_{cp}G_p + R_pX_mX_{cp}X_{cg}G_{cp}}{(R_p + X_{cp}^2G_p)^2 + X_p^2}\right]\right\} = 0 \quad (39)$$

From Eqs. (39) and (9) we write

$$N = \frac{X_mX_{cp}X_{cg}X_pG_{cp} - X_m^2X_{cp}^2G_p - X_m^2R_p}{(R_p + X_{cp}^2G_p)^2 + X_p^2} \quad (40)$$

Now the first term of Eq. (40) is the only term which may be positive. In this term X_{cp}, X_{cg}, and G_{cp} are necessarily positive. X_m may be positive or negative. $X_p = X_{Lp} - X_{cp}$ may also be positive or negative depending upon the frequency of the voltage E. If X_m is positive, X_p also must be positive for N to be positive. That is, the frequency of E must be above the natural frequency of the plate circuit. If M is negative, X_p also must be negative if resistance neutralization is to take place. That is, the frequency of E must be below the natural frequency of the plate tuned circuit. The equation thus checks the conclusion drawn from the vector diagram. As a further requirement for resistance neutralization we have

$$X_mX_{cp}X_{cg}X_pG_{cp} \text{ must be greater than } X_m^2X_{cp}^2G_p + X_m^2R_p \quad (41)$$

45. Triode Circuit with Coils in the Plate and Grid Circuits and a Condenser from the Plate to the Grid. The circuit diagram is shown by Fig. 39. The equations for this circuit are to be worked out and discussed in problem 10; so only the final results will be given here. When all of the resistance is considered as concentrated in the condenser branch, the equation in the standard form is

$$\mathbf{E} - \mathbf{I}_1\left\{R_1 - \frac{\mathfrak{X}_p\mathfrak{X}_g G_{cp} - \mathfrak{X}_p^2 G_p}{1 + (X_m G_{cp} + X_p G_p)^2} + j\left[X_n + \frac{(\mathfrak{X}_p\mathfrak{X}_g G_{cp} - \mathfrak{X}_p^2 G_p)(X_m G_{cp} + X_p G_p)}{1 + (X_m G_{cp} + X_p G_p)^2}\right]\right\} = 0 \qquad (42)$$

where

$$X_p = \omega L_p \qquad X_n = X_p + X_g - 2X_m - X_c$$
$$X_g = \omega L_g \qquad \mathfrak{X}_g = X_g - X_m$$
$$X_m = \omega M \qquad \mathfrak{X}_p = X_p - X_m$$
$$X_c = \frac{1}{\omega C}$$

The approximate values of U and δ are

$$U = \mathfrak{X}_p; \quad \delta = \frac{\mathfrak{X}_p}{\mathfrak{X}_g} = \frac{L_p - M}{L_g - M} \qquad (43)$$

M may be positive or negative.

46. Triode Circuit with Condensers in the Plate and the Grid Branches and a Coil between the Plate and Grid. The circuit diagram is shown by Fig. 27. The equations for this circuit are to be worked out and discussed in problem 15; so only the final results will be given here. The inductive bypasses are assumed not to affect the a.-c. equations, and all circuit losses are assumed as lumped in R. The final equation in the standard form is

$$\mathbf{E} - \mathbf{I}_1\left\{R - \frac{X_{cp}(X_{cg}G_{cp} - X_{cp}G_p)}{1 + X_{cp}^2 G_p^2} + j\left[X_L - X_{cg} - X_{cp} - \frac{(X_{cg}G_{cp} - X_{cp}G_p)X_{cp}^2 G_p}{1 + X_{cp}^2 G_p^2}\right]\right\} = 0 \qquad (44)$$

The approximate values of U and δ are

$$U = X_{cp} = \frac{1}{\omega C_p}; \quad \delta = \frac{X_{cp}}{X_{cg}} = \frac{C_g}{C_p} \qquad (45)$$

46a. Triode with an Auxiliary Circuit and a Tuned Grid Circuit.

The circuit is shown in Fig. 63*a*. Let the constants of the circuit be designated as shown on the diagram. The steady-state equations will now be derived by means of the complex algebra. As in the previous case, M_p and M_g represent the algebraic values and not the absolute values of the mutual inductances.

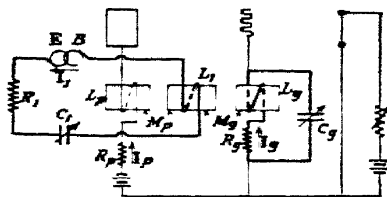



Fig. 63*a*—Triode with an auxiliary circuit and a tuned circuit in the grid branch.

A summation of the voltages around circuit 1 yields the equation

$$\mathbf{E} - R_1\mathbf{I}_1 - j\omega L_1\mathbf{I}_1 + \frac{j\mathbf{I}_1}{\omega C_p} - j\omega M_p\mathbf{I}_p - j\omega M_g\mathbf{I}_g = 0 \quad (46)$$

A like summation around the grid circuit yields

$$-j\omega M_g\mathbf{I}_1 - R_g\mathbf{I}_g - j\omega L_g\mathbf{I}_g + \frac{j\mathbf{I}_g}{\omega C_g} = 0 \quad (47)$$

The voltage of the grid is

$$\mathbf{E}_g = -\frac{j\mathbf{I}_g}{\omega C_g} \quad (48)$$

The plate voltage is

$$\mathbf{E}_p = -R_p\mathbf{I}_p - j(\omega M_p\mathbf{I}_1 + \omega L_p\mathbf{I}_p) \quad (49)$$

The plate current is given by the equation

$$\mathbf{I}_p = G_{cp}\mathbf{E}_g + G_p\mathbf{E}_p$$

$$= -G_{cp}\frac{j\mathbf{I}_g}{\omega C_g} + G_p(-R_p\mathbf{I}_p - j\omega M_p\mathbf{I}_1 - j\omega L_p\mathbf{I}_p)$$

$$\mathbf{I}_p = -\frac{j\left[\frac{G_{cp}\mathbf{I}_g}{\omega C_g} + \omega M_pG_p\mathbf{I}_1\right]}{D + j\omega L_pG_p} \quad (50)$$

in which

$$D = 1 + R_pG_p \quad (51)$$

Upon solving Eq. (47) for $\mathbf{I}_g$, we have

$$\mathbf{I}_g = -\frac{j\omega M_g\mathbf{I}_1}{\mathbf{Z}_g} \quad (52)$$

in which

$$\mathbf{Z}_g = R_g + j\left(\omega L_g - \frac{1}{\omega C_g}\right) \tag{53}$$

Substituting the value of $\mathbf{I}_g$ from Eq. (50),

$$\mathbf{I}_p = -\frac{\frac{M_g G_{cp}}{C_g \mathbf{Z}_g} + j\omega M_p G_p}{D + j\omega L_p G_p}\mathbf{I}_1 \tag{54}$$

Substituting the values of $\mathbf{I}_g$ and $\mathbf{I}_p$ from Eqs. (52) and (54) in Eq. (46), and solving,

$$\mathbf{E} = \mathbf{I}_1\left[\left\{R_1 + \omega^2\frac{M_p^2 G_p D}{W} + \frac{\omega^2}{Z_g^2}M_g^2 R_g - \frac{\omega X_g}{Z_g^2}\frac{SD}{W} - \frac{\omega}{Z_g^2}\frac{SR_gP}{W}\right\} + j\left\{X_1 - \omega^2\frac{M_p^2 G_p P}{W} - \frac{\omega^2 X_g}{Z_g^2}M_g^2 + \frac{\omega X_g}{Z_g^2}\frac{SP}{W} - \frac{\omega}{Z_g^2}\frac{SR_gD}{W}\right\}\right] \tag{55}$$

in which

$$D = 1 + R_p G_p$$
$$P = \omega L_p G_p$$
$$W = D^2 + P^2$$
$$S = \frac{M_g M_p G_{cp}}{C_g}$$
$$X_1 = \omega L_1 - \frac{1}{\omega C_1}$$
$$X_g = \omega L_g - \frac{1}{\omega C_g}$$
$$Z_g = \sqrt{R_g^2 + X_g^2}$$

In Eq. (55) the expressions for the resistance and the reactance each contain five terms. The first term is the resistance or reactance of circuit 1, the second term represents the effect of the current which flows in the plate circuit by reason of the plate conductance, the third term represents the effect of the alternating current circulating around the divided portion of the grid circuit, and the fourth and fifth terms represent the effect of the current in the plate circuit which is the result of the grid control.

The fourth term is the important term in the expression for the resistance. It changes sign as the impressed fre-

quency passes through the resonant frequency of the circulatory portion of the grid circuit, contributing a positive resistance on one side and a negative resistance on the other side of this resonant frequency. The first and fifth terms are the important terms of the expression for the reactance. The fifth term becomes large for frequencies near the resonant frequency of the grid circuit and may be used to increase the steepness of the curve between the resultant reactance of circuit 1 and the impressed frequency.

To obtain a better idea of the relations expressed in Eq. (55), let us apply this equation to a particular circuit by plotting the resistance and reactance of the circuit as functions of the frequency of the alternating electromotive force impressed in circuit 1. Let the coupling be as represented in Fig. 63*a*; that is, M_p is positive and M_g is negative. The constants of the circuit with the exceptions noted below are to be the same as given in Sec. 14 for Fig. 25. The exceptions are

$$R_1 = 66 \text{ ohms}$$
$$M_g = -35.7 \text{ microhenrys}$$
$$C_g = 16.6 \text{ microfarads}$$
$$\omega_{rg} = 1.8715 \times 10^5 \text{ radians per second}$$

The curves of Fig. 63*b* show how the net resistance of circuit 1 varies with the impressed frequency, while those of Fig. 63*c* show the variation of the reactance with the frequency. An examination of these curves shows that the circuit constants have been proportioned so that the net resistance is near the minimum value at the angular velocity of 1.84×10^5 radians per second, at which the net reactance is zero. It will be seen that the slope of the curve for the net reactance is much greater near this angular velocity than is the slope of the curve X_1, showing the reactance of circuit 1 without the neutralizer associated with it. This means that the selective coefficient against frequencies detuned by a few per cent is much greater than for a Fig. 25 connection in which pure resistance neutralization is alone utilized.

The selective coefficients, against frequencies detuned by 1 per cent, or more, of a circuit having the constants of Fig. 41 have been plotted in Fig. 63*d* for the following conditions:

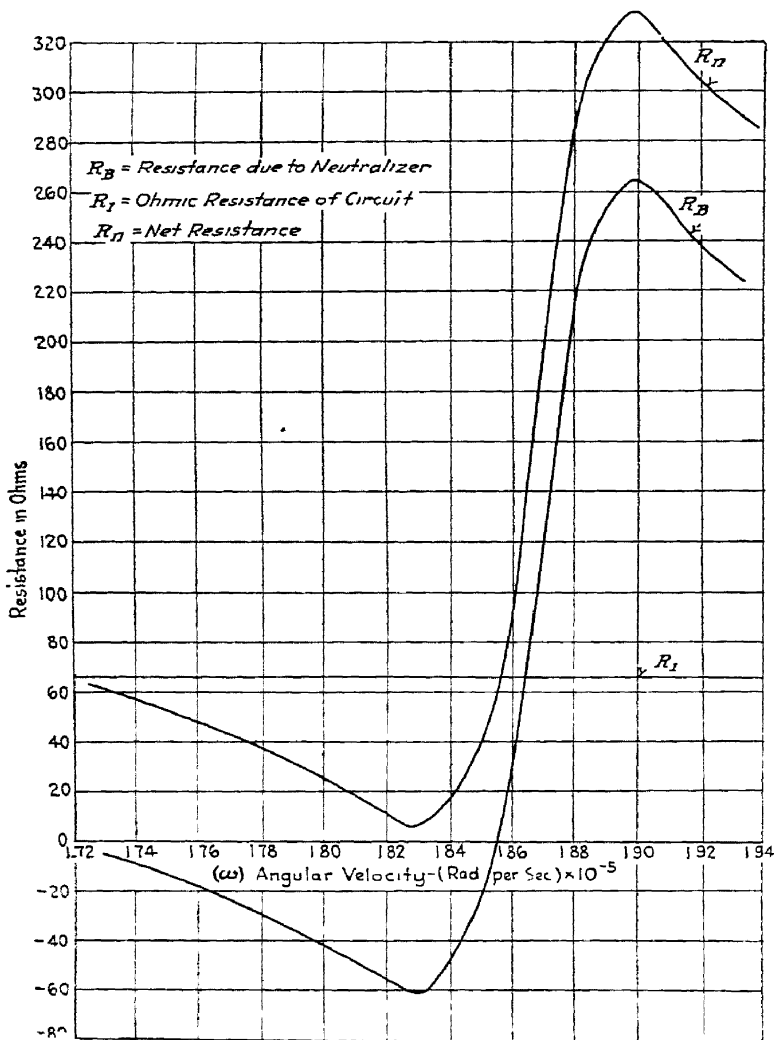



FIG 63*b*.—Relation between resistance and impressed frequency.

The steady-state selective coefficient is given by the expression

$$S_c = \frac{Z_i^2}{Z_c^2}$$

If the correspondent station has a wave length corresponding to 183,500 radians per second, $Z_e = 6$ ohms. Z_s for any other frequency can be found by combining the resistance values given by curve R_s of Fig. 63*b* and the reactance values given by curve X_s of Fig. 63*c*. The manner in

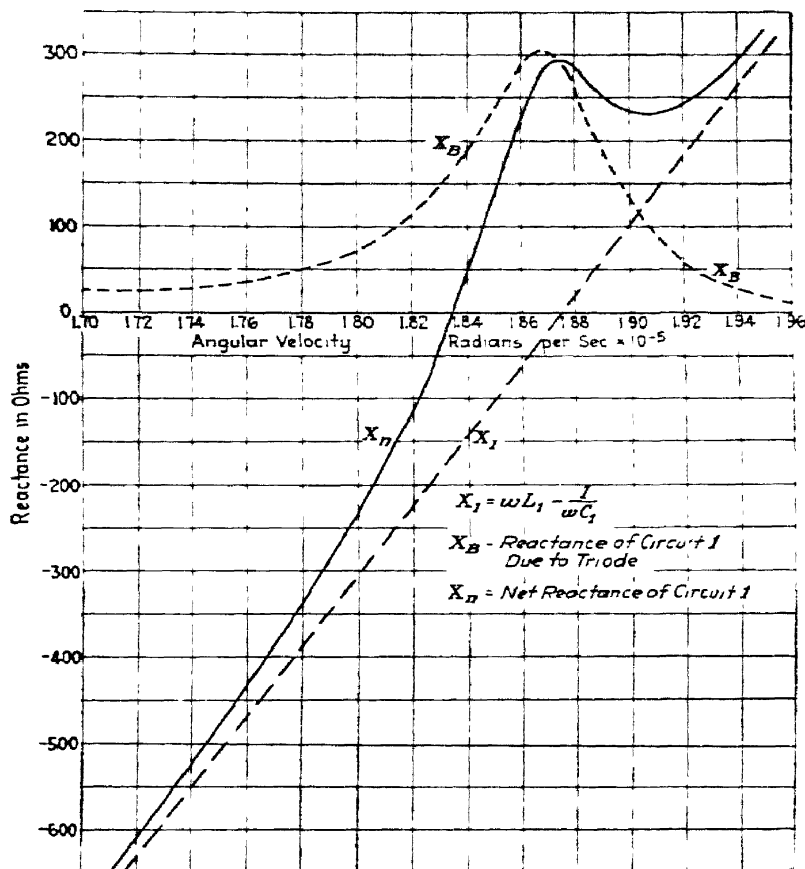



FIG 63*c*.—Relation between reactance and impressed frequency.

which the selective coefficient varies with the interferent frequency in this circuit when the correspondent has a wave length corresponding to an angular velocity of 183,500 radians per second is shown by curve *A* of Fig. 63*d*.

Curve *B* shows the selective coefficient of the same power circuit for the case in which the neutralizer circuit is adjusted (as in Fig. 25) to give pure resistance neutralization,

reducing the net resistance of the circuit to 6 ohms. For curves *B* and *C* the angular velocity of the correspondent station was taken as 187,500 radians per second. Curve *C* shows the selective coefficient of circuit 1 without resistance neutralization, its net resistance at the resonant frequency being $R_1 = 66$ ohms. The increased selectivity against

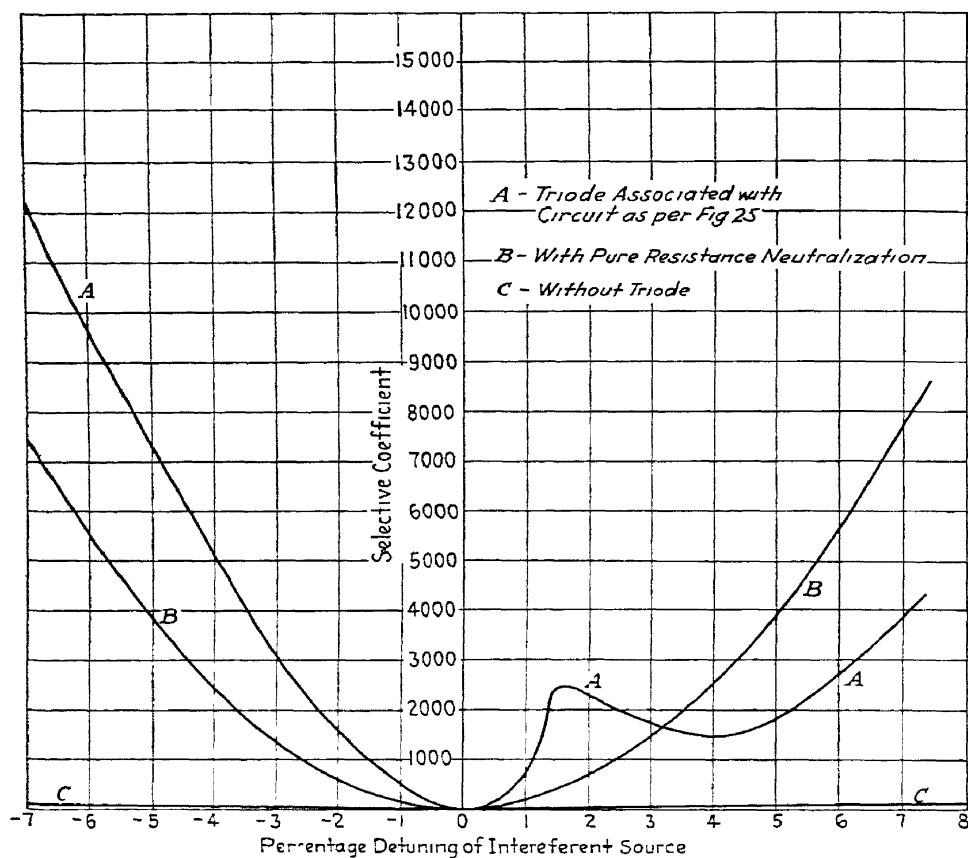
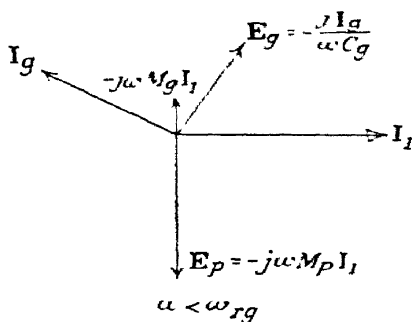
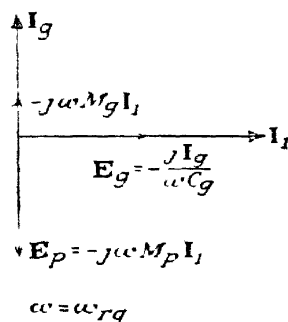
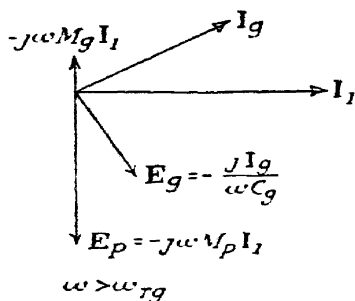



FIG 63*d*.—Steady state selective coefficients.

frequencies detuned by 1 or 2 per cent which results from the rapid increase of the reactance and the addition of resistance at frequencies slightly removed from the resonant frequency is illustrated by these curves.

In order to bring out the phase relations existing between the various currents and voltages of the Fig. 63*a* circuit, approximate vector diagrams have been drawn for three

conditions. These diagrams are approximate in that the effect of the plate space current upon the plate voltage has been neglected.

Case I.—The generator frequency is below the natural frequency of the grid oscillatory circuit. The vector diagram for this case is shown by Fig. 63*e*. The current in the generator circuit $\mathbf{I}_1$ is taken as a reference vector. The impressed voltage in the grid circuit is the voltage induced

FIGS. 63*e*, *f* and *g*.—Approximate vector diagrams for the circuit of Fig. 63*a*. Cases for which M_p is positive and M_g is negative.

by $\mathbf{I}_1$ in the grid coil and is equal to $-j\omega M_g \mathbf{I}_1$. Since M_g is negative, this voltage leads the current $\mathbf{I}_1$ by 90 degrees. Since the generator frequency is lower than the natural frequency of the grid tuned circuit, this circuit has a net condensive reactance. Therefore the circulating current $\mathbf{I}_g$ leads the voltage impressed in the grid tuned circuit and on the diagram $\mathbf{I}_g$ is shown as leading the voltage $-j\omega M_g \mathbf{I}_1$. The grid voltage is the voltage of the condenser C_g in the

direction opposite to the arrow direction. The grid voltage therefore is $\frac{-j\mathbf{I}_g}{\omega C_g}$; that is, it lags $\mathbf{I}_g$ by 90 degrees. It is so shown on the diagram. The approximate plate voltage is $-j\omega M_p \mathbf{I}_1$, where M_p is positive. This voltage lags I_1 by 90 degrees and is so shown on the diagram. Since the plate and grid voltages are separated in phase by more than 90 degrees, the phase positions are correct for resistance neutralization. This checks the results given by the curves of Fig. 63*b*.

Case II.—The generator frequency and the frequency of the grid tuned circuit are the same. The vector diagram for this case is shown by Fig. 63*f*. For this case the net grid oscillatory circuit reactance is zero, and the current I_g is therefore in phase with the voltage $-j\omega M_g I_1$. The vectors therefore have the positions shown on the diagram. Since the plate and grid voltages are just 90 degrees out of phase, no resistance neutralization takes place.

Case III.—The generator frequency is higher than the natural frequency of the grid tuned circuit. The vector diagram for this case is shown by Fig. 63*g*. The net grid tuned circuit reactance for this case is inductive and the current I_g therefore lags behind the voltage induced in the grid coil. The vectors therefore have the relative phase positions shown on the diagram. The plate and grid voltages are separated by less than 90 degrees; so the triode absorbs power, that is, it adds positive resistance to the circuit 1. This checks the conclusions drawn from the curves of Fig. 63*b*.

For the particular constants given for the circuit of Fig. 41, the reduction factor γ at resonance is

$$\gamma = \frac{R - N}{R} = \frac{6}{66} = 0.091$$

The increase in the abstractive coefficient due to the presence of the neutralizer $\left(\text{equal to } \frac{1}{\gamma}\right) = 11$. The regenerative amplification is $\left(\frac{1}{\gamma}\right)^2 = 121$.

The data given for this circuit are not typical of the constants of a radio receiving circuit, as γ for a radio receiving circuit would be much less than 0.091. The discussion of this circuit, however, illustrates very well the general method of treating systems of this nature.

47. Generalized Amplifier Circuit.

The circuit to be treated in this section is shown in Fig. 64. As the heading of the section indicates, this circuit may be used as a basis for the discussion of many amplifier circuits. The equations derived in this section will be

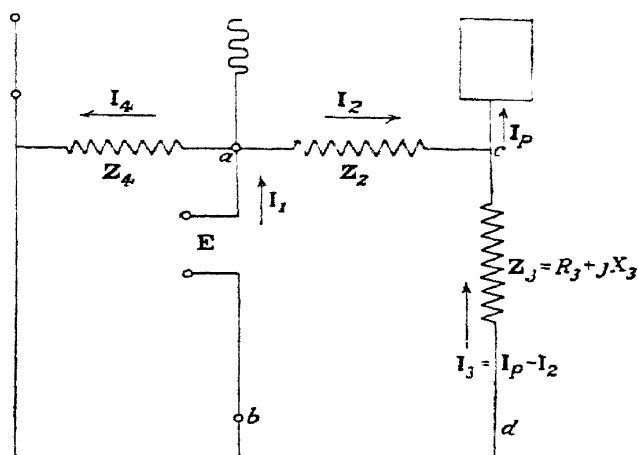



FIG 64.—Generalized amplifier circuit.

used in many of the later sections to build up solutions for circuits in which the internal capacitances of the triode are taken into account. Thus in some cases $\mathbf{Z}_2$ may stand for nothing but the internal capacity reactance from plate to grid. In all cases the internal capacity reactance must enter into the value of $\mathbf{Z}_2$. The impedance $\mathbf{Z}_4$ must include the internal capacity reactance from grid to filament, and $\mathbf{Z}_3$ must include the internal capacity reactance from plate to filament.

If we apply Kirchoff's current law at the junction a, we obtain the equation

$$\mathbf{I}_1 = \mathbf{I}_4 + \mathbf{I}_2;\ \mathbf{I}_2 = \mathbf{I}_1 - \mathbf{I}_4 \tag{56}$$

By equating the sum of the e.m.fs. around the loop *bacd* to zero, we obtain

$$\mathbf{E} - \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_3(\mathbf{I}_p - \mathbf{I}_2) = 0 \tag{57}$$

The plate and grid potentials are given by the equations

$$\mathbf{E}_g = \mathbf{E};\ \mathbf{E}_p = -\mathbf{Z}_3(\mathbf{I}_p - \mathbf{I}_2) \tag{58}$$

The plate space current is given by the equation

$$\begin{aligned} \mathbf{I}_p &= \mathbf{E}G_{cp} - \mathbf{Z}_3G_p(\mathbf{I}_p - \mathbf{I}_2) \\ &= \frac{\mathbf{E}G_{cp}}{1 + \mathbf{Z}_3G_p} + \frac{\mathbf{Z}_3G_p\mathbf{I}_2}{1 + \mathbf{Z}_3G_p} \end{aligned} \tag{59}$$

Substituting Eq. (59) in Eq. (57) and collecting terms gives

$$\mathbf{E}\left(1 + \frac{\mathbf{Z}_3G_{cp}}{1 + \mathbf{Z}_3G_p}\right) - \mathbf{I}_2\left[\mathbf{Z}_2 + \mathbf{Z}_3 - \frac{\mathbf{Z}_3^2G_p}{1 + \mathbf{Z}_3G_p}\right] = 0 \tag{60}$$

$$\mathbf{E} - \mathbf{I}_2\left[\frac{\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3G_p}{1 + \mathbf{Z}_3(G_p + G_{cp})}\right] = 0 \tag{61}$$

The current $\mathbf{I}_4$ is given by

$$\mathbf{I}_4 = \frac{\mathbf{E}}{\mathbf{Z}_4} \tag{62}$$

Upon substituting Eqs. (62) and (56) in Eq. (61), there results

$$\mathbf{E} - \mathbf{I}_1\left[\frac{\mathbf{Z}_4[\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3G_p]}{\mathbf{Z}_4[1 + \mathbf{Z}_3(G_p + G_{cp})] + \mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3G_p}\right] = 0 \tag{63}$$

The bracketed term of Eq. (63) is the impedance which would be measured on a bridge connected across the terminals marked *E*. That is, the bracketed term of Eq. (63) is the input impedance of the circuit. We shall represent this impedance by the symbol $\mathbf{Z}_{ab}$; that is,

$$\mathbf{Z}_{ab} = \frac{\mathbf{Z}_4[\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3G_p]}{\mathbf{Z}_4[1 + \mathbf{Z}_3(G_p + G_{cp}) + \mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3G_p} \tag{64}$$

In order to obtain an expression for the current $\mathbf{I}_3$, we write Kirchoff's e.m.f. law around the loop *bacd* in the form

$$\mathbf{E} - \mathbf{Z}_2(\mathbf{I}_p - \mathbf{I}_3) + \mathbf{Z}_3\mathbf{I}_3 = 0 \tag{65}$$

Since the plate potential is given by the expression

$$\mathbf{E}_p = -\mathbf{Z}_3\mathbf{I}_3$$

the plate current is

$$\mathbf{I}_p = \mathbf{E}G_{cp} - \mathbf{Z}_3\mathbf{I}_3G_p \tag{66}$$

Upon substituting Eq. (66) in Eq. (65) there results

$$\mathbf{E} - \mathbf{I}_3\left[-\frac{\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3G_p}{1 - \mathbf{Z}_2G_{cp}}\right] = 0 \tag{67}$$

Equation (67) determines the current $\mathbf{I}_3$. The power expended in the output impedance Z_3 is

$$P = I_3^2R_3 \tag{68}$$

Problems

15. Work out and discuss the equations for the circuit shown by Fig. 27 What conditions must be fulfilled in order to obtain resistance neutralization? Draw the approximate vector diagram for the circuit and with the aid of this diagram show how the conditions for power output are satisfied Assume that the inductive bypass coils do not affect the alternating currents and voltages.

16. In the circuit of Fig. 55 the condenser C_1 represents the antenna of a radio receiving station, and the generator represents the voltage induced in the antenna by the impinging waves. The waves of the correspondent station have an angular velocity of 10^6 radians per second, and the circuit is adjusted so that the total reactance is zero to this angular velocity The tube is a 201-A radiotron. Its constants are given in the table of triode constants at the end of Chap. I In this circuit

$L_p = 4 \times 10^{-4}$ henrys;	$R_p = 40$ ohms
$M = -10^{-4}$ henrys;	$C_1 = 2 \times 10^{-9}$ farads
$R_1 = 36\ 5$ ohms	

What value must L_g have in order to make the net reactance of the system to the correspondent frequency equal to zero? What is the value of X and of X_4? What is the reduction factor? What is the value of the regenerative amplification at resonance? What is the value of the steady-state selective coefficient against a station detuned by 2 per cent?

CHAPTER VII

MODULATION AND DEMODULATION

48. Preliminary Considerations.

The preceding chapters have dealt with applications of the triode in which it was desirable to locate the operating point on a plane or nearly plane portion of the characteristic surface. The equations which have been derived were obtained by assuming a linear relation to exist between the impressed variable potentials and the resulting variable currents. In discussing the generation of sustained oscillations, the definitions of the triode constants were modified in such a manner as to take into account approximately the curvature of the characteristic curves. We now proceed to consider some applications of the triode which depend entirely upon the curvature of the characteristic curves. These applications are demodulation and grid circuit modulation in carrier-current communication systems. The methods which have been used up to this time cannot be employed to treat these applications. Before we can proceed to a discussion of modulation and demodulation, the characteristic surface of the triode must be expressed in the form of an equation.

As stated in Chap. I, the equation of the characteristic surface of a triode may be expressed in the form

$$i_{pt} = f(\mu v_g + v_p) = f(\alpha) \qquad (1)$$

Where i_{pt} is the total plate space current, v_g is the total grid voltage, v_p is the total plate voltage, and

$$\alpha \text{ represents } (\mu v_g + v_p) \qquad (2)$$

Let us now consider the triode circuit in Fig. 12. Let the continuous grid voltage be represented by E_{gp} and the

continuous plate voltage by E_{pp}. Then before the alternators are started, the plate space current is given by

$$I_{pp} = f(\mu E_{gp} + E_{pp}) = f(\alpha_p) \tag{3}$$

where α_p represents $(\mu E_{gp} + E_{pp})$. Now let the alternators be started. Let these alternators deliver voltages having any wave form. Let the instantaneous value of the voltage delivered by the grid alternator be e_g and that of the plate alternator be e_p. Then, from Eq. (1) we have

$$i_{pt} = f(\alpha) = f[\mu(E_{gp} + e_g) + (E_{pp} + e_p)] \tag{4}$$

If we expand $f(\alpha)$ about the point α_p in a Taylor series, we obtain

$$f(\alpha) = f(\alpha_p) + a_1(\alpha - \alpha_p) + a_2(\alpha - \alpha_p)^2 + \cdots \tag{5}$$

where

$$a_n = \frac{1}{n!}\left(\frac{d^n f(\alpha)}{d\alpha^n}\right)_{\alpha=\alpha_p} \tag{6}$$

But

$$\frac{d^n}{d(\mu v_g + v_p)^n}[f(\mu v_g + v_p)] = \frac{\partial^n f(\alpha)}{\partial v_p^n} = \frac{1}{\mu^n}\frac{\partial^n f(\alpha)}{\partial v_g^n} \tag{7}$$

So Eq. (5) becomes

$$i_{pt} = I_{pp} + a_1(\mu e_g + e_p) + a_2(\mu e_g + e_p)^2 + \cdots \tag{8}$$

where a_n may be written

$$a_n = \frac{1}{n!}\left(\frac{\partial^n i_{pt}}{\partial v_p^n}\right)_{\alpha=\alpha_p} = \frac{1}{n!\mu^n}\left(\frac{\partial^n i_{pt}}{\partial v_g^n}\right)_{\alpha=\alpha_p} \tag{9}$$

The variable plate current is

$$i_p = i_{pt} - I_{pp} = a_1(\mu e_g + e_p) + a_2(\mu e_g + e_p)^2 + \cdots \tag{10}$$

Equation (10) is the general form of the expansion of the variable plate current. If the characteristic curves are straight lines, we see from Eq. (9) that

$$a_1 = G_p$$
$$a_2 = a_3 = a_4 = \cdots a_n = 0$$

and we have our familiar equation

$$i_p = G_p(\mu e_g + e_p) = G_{cp}e_g + G_p e_p \tag{11}$$

If we replace the alternator in the plate circuit by an impedance, then the only variable plate voltage is that due to the plate current flowing through the impedance, and we may write

$$e_p = F_1(i_p) \tag{12}$$

and the expression for i_p may be written as

$$i_p = f(\mu e_g + F_1(i_p)) \tag{13}$$

If Eq. (13) is solved for i_p, we obtain

$$i_p = F(e_g) \tag{14}$$

That is, when there is only an impedance in the plate circuit, the variable plate current is a function of the grid voltage alone. If we expand Eq. (14) in a Taylor series, we obtain

$$i_p = b_1 e_g + b_2 e_g^2 + b_3 e_g^3 + \cdot \cdot \cdot \tag{15}$$

It is possible to determine the value of the coefficients b_1, b_2, etc. for any general impedance in the plate circuit. Only the case in which the impedance is a pure resistance, however, will be discussed here. For this case

$$e_p = F_1(i_p) = -R_p i_p \tag{16}$$

If we substitute Eq. (16) in Eq. (10), there results

$$i_p = a_1(\mu e_g - R_p i_p) + a_2(\mu e_g - R_p i_p)^2 + \cdot \cdot \cdot \tag{17}$$

If we substitute i_p as given by Eq. (15) in both sides of Eq. (17), we obtain on both sides of the new equation a series in powers of e_g. The value of the coefficients b_1, b_2, etc. may be determined from this equation by equating coefficients of like powers of e_g. Upon doing this we obtain for b_1 and b_2

$$b_1 = \frac{a_1 \mu}{1 + a_1 R_p} \tag{18}$$

$$b_2 = \frac{a_2 \mu^2}{(1 + a_1 R_p)^3} \tag{19}$$

The characteristic curves of most triodes are of such a nature that only the first two terms of Eq. (15) need be used for practical work.

From Eq. (9)

$$a_1 = G_p = \frac{G_{cp}}{\mu} \tag{20}$$

where G_p is to be taken for a very small increment in plate voltage about the operating point and G_{cp} is to be taken for a very small variation in grid voltage about the operating point.

Let us now take the plate current–grid voltage characteristic of our triode for a plate voltage of E_{pp} and plot the square root of the plate current as ordinates and the grid voltage as abscissas. If the permanent plate and grid voltages are properly chosen and if the grid voltage variation is properly limited, this curve will be essentially a straight line. The equation of this straight line will be

$$\sqrt{i_{pp}} = A + Be_g \tag{21}$$

where A is the intercept on the current axis and B is the slope of the straight line. Upon squaring both sides of Eq. (21), we obtain

$$i_{pp} = A^2 + 2ABe_g + B^2e_g^2 \tag{22}$$

$$\frac{\partial^2 i_{pp}}{\partial e_g^2} = 2B^2 \tag{23}$$

From Eqs. (9) and (23) we write

$$a_2 = \frac{1}{2}\frac{1}{\mu^2}2B^2 = \frac{B^2}{\mu^2} \tag{24}$$

Upon substituting Eqs. (20) and (24) in Eqs. (18) and (19), there results

$$b_1 = \frac{G_{cp}}{1 + G_pR_p} \tag{25}$$

$$b_2 = \frac{B^2}{(1 + G_pR_p)^3} \tag{26}$$

In these equations the constants G_{cp}, G_p, and B are to be determined according to the methods given in the preceding two paragraphs.

Figure 65 gives the plate current–grid voltage curve, (plate potential 20 volts) for a 201-A tube. On the same sheet the square root of the plate current is plotted against

the grid voltage. If the operating point is taken as $E_{gp} = +1$ and $E_{pp} = +20$, the grid voltage may vary from -1 to $+3$ volts, and the square-root curve will be essentially a straight line. For this tube when operating at the point given above, $B = 5.75 \times 10^{-3}$. $G_{cp} = (700 - 500) \times$

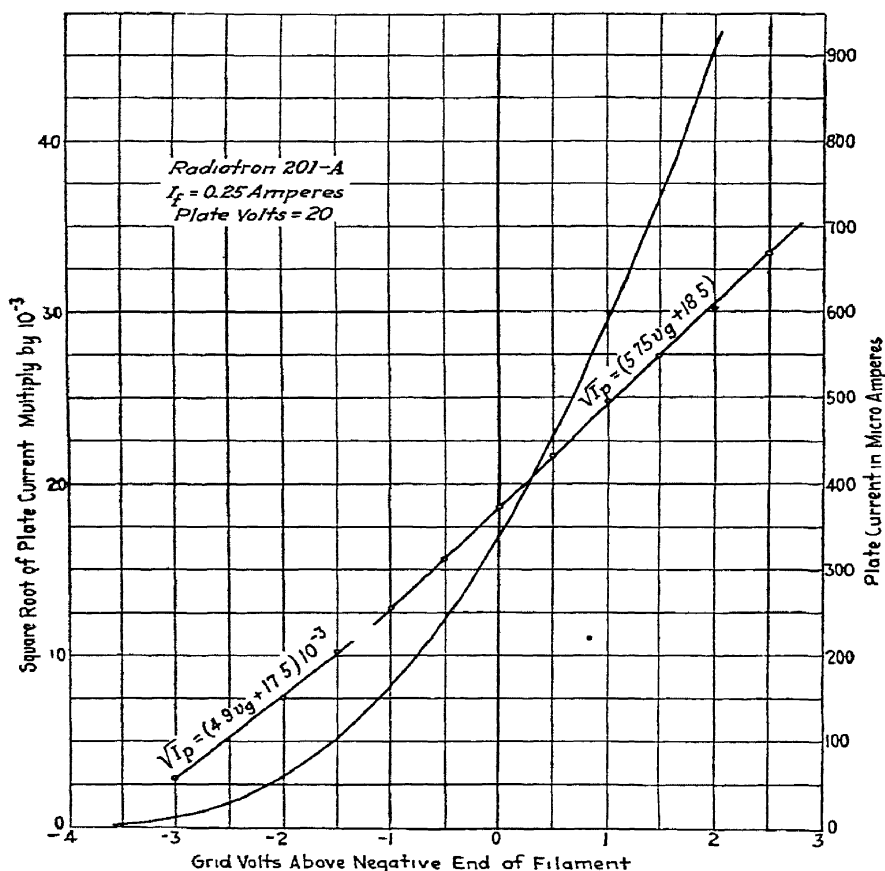



Fig 65.

$10^{-6} \div (1.4 - .7) = 286 \times 10^{-6}$. If $R_p = \dfrac{1}{5G_p}$, then for the 201-A tube when operation takes place at the point given above

$$b_1 = \frac{286 \times 10^{-6}}{1.2} = 238 \times 10^{-6}$$

$$b_2 = \frac{(5.75 \times 10^{-3})^2}{(1.2)^3} = 19.15 \times 10^{-6}$$

and Eq. (15) is

$$i_p = 238 \times 10^{-6} e_g + 19.15 \times 10^{-6} e_g^2$$

49. Grid Circuit Modulation.

Consider the circuit shown schematically by Fig. 66. A voltage due to the voice is impressed across the terminals marked e_v while the voltage due to a high-frequency generator, such as a triode oscillator, is impressed across the terminals marked e_c. The voltage due to the voice is considered as broken up into its component frequencies, and only one

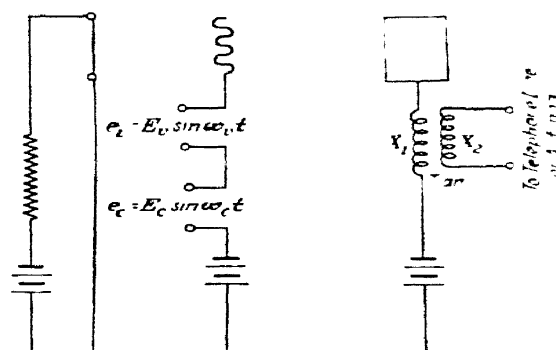



FIG. 66.—Grid circuit modulator.

of these frequencies is considered. The antenna or telephone line is associated with the output circuit by means of a transformer, and the adjustments are assumed to be such that the whole system may be replaced in the plate circuit by a pure resistance R_p. The operating point of the triode is so chosen that the first two terms of Eq. (15) give a close approximation to the variable plate current.

The variable grid voltage is

$$e_g = e_v + e_c = E_v \sin \omega_v t + E_c \sin \omega_c t \qquad (27)$$

Upon substituting Eq. (27) in Eq. (15) and using only the first two terms of the latter equation, we obtain

$$\begin{aligned} i_p = b_1 E_v \sin \omega_v t &+ b_1 E_c \sin \omega_c t \\ &+ b_2 E_v^2 \sin^2 \omega_v t + b_2 E_c^2 \sin^2 \omega_c t \\ &+ 2 b_2 E_v E_c (\sin \omega_v t)(\sin \omega_c t) \qquad (28) \end{aligned}$$

In radio communication the antenna tuning and the functioning of the transformer are such that only the second and last terms of Eq. (28) are of appreciable magnitude in the antenna current. If the transformer functions essentially as an ideal transformer for these frequencies and if the antenna is properly tuned, then the antenna current will be

$$i_a = pb_1E_c \sin \omega_c t + 2pb_2E_cE_v (\sin \omega_v t) (\sin \omega_c t) \quad (29)$$

where p is the voltage transformation ratio of the output transformer; that is,

$$p^2 = \frac{X_1}{X_2} \quad (30)$$

If to Eq. (29) we apply the formula

$$2(\sin \alpha)(\sin \beta) = \cos (\alpha - \beta) - \cos (\alpha + \beta) \quad (31)$$

we obtain

$$i_a = pb_1E_c \sin \omega_c t + pb_2E_cE_v \cos (\omega_c - \omega_v)t - pb_2E_cE_v \cos (\omega_c + \omega_v)t \quad (32)$$

The first term of Eq. (32) is called the **carrier wave,** the second wave the **lower side band,** and the third term the **upper side band.** The last two terms are called side bands because ω_v has a band of values such that $\frac{\omega_v}{2\pi}$ ranges from 0 to 2,000 cycles per second. These two terms thus represent a band of frequencies 2,000 cycles wide lying one below the carrier frequency and one above the carrier frequency. For good speech transmission the antenna must have very low reactance to a band of frequencies 4,000 cycles wide centered on the carrier frequency.

To obtain a picture of the antenna current we write Eq. (29) in the form

$$i_p = \sin \omega_c t[pb_1E_c + 2pb_2E_cE_v \sin \omega_v t] \quad (33)$$

This equation represents a sine wave having the angular velocity ω_c and having a variable amplitude given by the term in the brackets. Such a current is sketched in Fig.

67. If the modulation is to be complete, that is, if the amplitude of the carrier frequency is to be reduced to zero once for each cycle of the voice frequency, then

$$b_1 \text{ must equal } 2b_2E_v$$

That is, the voice voltage impressed on the grid must have an amplitude given by

$$E_v = \frac{b_1}{2b_2} = \frac{G_{cp}(1 + R_pG_p)^2}{2B^2} \tag{34}$$

If R_a is the antenna resistance, the average power in either side band is

$$P = \frac{p^2b_2^2E_c^2E_v^2R_a}{2} \tag{35}$$

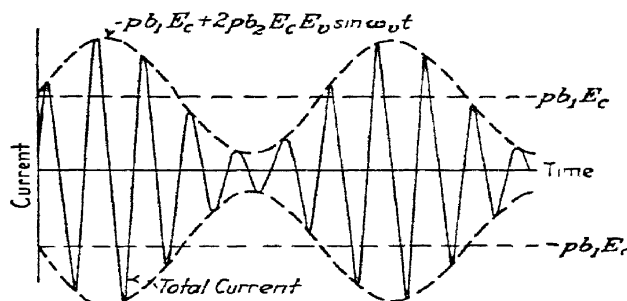



FIG 67.—Modulated current wave

But from the transformer theory developed in Chap. II, $p^2R_a = R_p$. If we substitute this value and also the value of b_2 as given by Eq. (26) in Eq. (35), we obtain

$$P = \frac{B^4E_c^2E_v^2R_p}{2(1 + R_pG_p)^6} \tag{36}$$

We can find the value of R_p which makes this a maximum by taking the derivative of P with respect to R_p, equating this derivative to zero, and solving for R_p. Upon carrying out these operations, we find that the best value for R_p is

$$R_p \text{ (maximum power in side bands)} = \frac{1}{5G_p} \tag{37}$$

The output transformer should be so designed that

$$p^2R_a = \frac{X_1}{X_2}R_a = \frac{1}{5G_p}$$

That is, the impedance transformation ratio of the transformer should be (see Sec. 8)

$$\frac{X_1}{X_2} = p^2 = \frac{1}{5R_aG_p} \tag{38}$$

From Eq. (32) we see that the amplitude of the side bands contains b_2 as a factor. When the output resistance has the best possible value as given by Eq. (37), then from Eq. (26)

$$b_2 = \frac{B^2}{(1 + \frac{1}{5})^3} = \frac{B^2}{(1.2)^3} = \frac{B^2}{1.73} \tag{39}$$

That is, B, the slope of the $\sqrt{i_{pp}}$ *vs.* v_g curve, is a measure of the value of the tube as a grid circuit modulator. If B were zero, no modulation would take place.

50. Demodulation without a Grid Stopping Condenser.

In the system now to be considered, a voltage due to the antenna current is impressed in the grid circuit of the triode, and a utilization device such as a telephone receiver or an audio frequency transformer feeding an audio frequency amplifying system is inserted in the plate circuit. The device in the plate circuit is assumed to be replaceable, so far as the operation of the triode is concerned, by a resistance R_p. Such a circuit is shown schematically by Fig. 68. The operating point is again so chosen that the first two terms of Eq. (15) give a close approximation to the plate space current.

FIG 68 —Demodulator circuit

Now the current in the receiving antenna will have the same form as the current in the transmitting antenna, and therefore the voltage impressed on the grid of the demodulator will be given by an equation of the form of Eq. (32). Therefore the grid variable voltage will be taken as

$$e_g = k_1 \sin \omega_c t + k_2 \cos (\omega_c - \omega_v)t - k_2 \cos (\omega_c + \omega_v)t \tag{40}$$

The current in the output circuit then is

$$i_p = b_1 e_g + b_2[k_1^2 \sin^2 w_c t + k_2^2 \cos^2 (\omega_c - \omega_v)t + k_2^2 \cos^2 (\omega_c + \omega_v)t] + 2b_2 k_1 k_2 (\sin \omega_c t)[\cos (\omega_c - \omega_v)t] - 2b_2 k_1 k_2 (\sin \omega_c t)[\cos (\omega_c + \omega_v)t] - 2b_2 k_2^2 [\cos (\omega_c - \omega_v)t][\cos (\omega_c + \omega_v)t] \quad (41)$$

The first term of Eq. (41) contains the same frequencies as e_g. The second term can be broken up into a direct-current component plus three currents whose frequencies are respectively $2f_c$, $2(f_c - f_v)$, and $2(f_c + f_v)$. The other three terms reduce as follows:

$$2b_2 k_1 k_2 (\sin \omega_c t)[\cos (\omega_c - \omega_v)t] = b_2 k_1 k_2 [\sin (2\omega_c - \omega_v)t + \sin \omega_v t] \quad (42)$$

$$2b_2 k_1 k_2 (\sin \omega_c t)[\cos (\omega_c + \omega_v)t] = b_2 k_1 k_2 [\sin (2\omega_c + \omega_v)t - \sin \omega_v t] \quad (43)$$

$$2b_2 k_2^2 [\cos (\omega_c - \omega_v)t][\cos (\omega_c + \omega_v)t] = b_2 k_2^2 [\cos 2\omega_c t + \cos 2\omega_v t] \quad (44)$$

The plate space current can then be written in the form i_p = (direct-current component) + (component containing the frequencies f_c, $f_c + f_v$, $f_c - f_v$, $2f_c$, $2(f_c - f_v)$, $2(f_c + f_v)$, $2f_c - f_v$, $2f_c + f_v$), $+2b_2 k_1 k_2 \sin \omega_v t - b_2 k_2^2 \cos 2\omega_v t$ (45)

Of these terms the first two either do not affect the receiving apparatus or may be removed with filter networks. The third term gives us the useful voice frequencies. The fourth term is a distortion term. This term would drop out if only one side band were transmitted.

The power in the voice currents is

$$P_v = b_2^2 k_1^2 k_2^2 R_p \quad (46)$$

Since this contains R_p in the same way as Eqs. (35) and (36), the power in the voice frequencies will be a maximum when

$$R_p = \frac{1}{5G_p} \quad (37)$$

Since the amplitude of the voice frequency current contains b_2 as a factor and since the best value of b_2 is given by Eq. (39), it is seen that B, the slope of the $\sqrt{i_{pp}}$ *vs.* v_g curve, is also a measure of the value of a triode when used as a demodulator without a grid condenser.

51 and 52. Plate Circuit Modulation.

The modulation of a high-frequency current f_c by a lower frequency f_v consists in making the amplitude of the high-frequency current vary along a sine wave having the frequency f_v. Any system which will cause this variation in the amplitude of the high-frequency current may be used as a modulator. There are many schemes for using triode circuits as modulators. The grid circuit modulating system studied in Sec. 49 depends for its action upon the fact that the plate space current is a non-linear function of the grid voltage. In the system about to be described, a different principle is involved. If the continuous plate potential of a triode generator of sustained oscillations is varied, the amplitude of the oscillating current also varies. Therefore if we vary the plate potential of such an oscillator at a low frequency, the amplitude of the high-frequency oscillations will vary at the low frequency. The ideal relation between the change in amplitude of the high-frequency oscillations and the change in continuous plate potential is a linear one. It has been found by experiment that this ideal relation holds very closely for a considerable range of plate potentials; that is, if I_c is the amplitude of the oscillating current, then

$$\Delta I_c = k\Delta E_{pp} \tag{47}$$

Let the voltage in the plate circuit due to the voice be

$$e_v = E_v \sin \omega_v t \tag{48}$$

To the high frequency oscillations, this voice voltage is equivalent to a change in the continuous plate potential; so we write

$$\Delta E_{pp} = E_v \sin \omega_v t \tag{49}$$

Let the oscillating current before the transmitter is spoken into be

$$i_c = I_c \sin \omega_c t \tag{50}$$

When the transmitter is operating, the oscillating current becomes

$$i_c = (I_c + \Delta I_c) \sin \omega_c t \tag{51}$$

Upon substituting Eqs. (47) and (49) in Eq. (51), there results

$$i_c = I_c \sin \omega_c t + kE_v(\sin \omega_c t)(\sin \omega_v t) \tag{52}$$

Since Eq. (51) has the same form as Eq. (29), all of the general discussion of Eq. (29) which depended upon the form of the equation applies also to Eq. (51) and will not be repeated here.

The modulation is complete in this system when

$$kE_v = I_c \tag{53}$$

If E_{pp} is the steady plate voltage, it has been found that k is given roughly by the relation

$$k = \frac{I_c}{E_{pp}} \tag{54}$$

Now the theory of Chap. IV gives us a means of determining I_c when the circuit constants and the characteristic curves of the triode are available. Therefore the theory of Chap. IV together with Eq. (54) enable us to estimate the value of k for any ordinary oscillating system.

From Eqs. (53) and (54) we see that if the modulation is complete,

$$E_v = E_{pp} \tag{55}$$

The power supplied by the generator which furnishes the plate continuous potential is $E_{pp}I_{pp}$. But I_{pp} is also the peak value of the alternating current which flows at voice frequency when Eq. (55) holds. Therefore the power which must be supplied by the voice is $\frac{1}{2}E_{pp}I_{pp}$, or half as much as is furnished by the plate direct current generator. Since the efficiency of conversion from direct current power

to alternating current power is not much in excess of 50 per cent, it follows that to modulate completely a high-frequency current by this system, the power in the voice frequency currents must equal the power in the high-frequency currents. For this reason the transmitter must be associated with an amplifier, and the output voltage of the amplifier is introduced into the plate circuit of the oscillator tube. The power output of the amplifier must equal the power in the high-frequency oscillations.

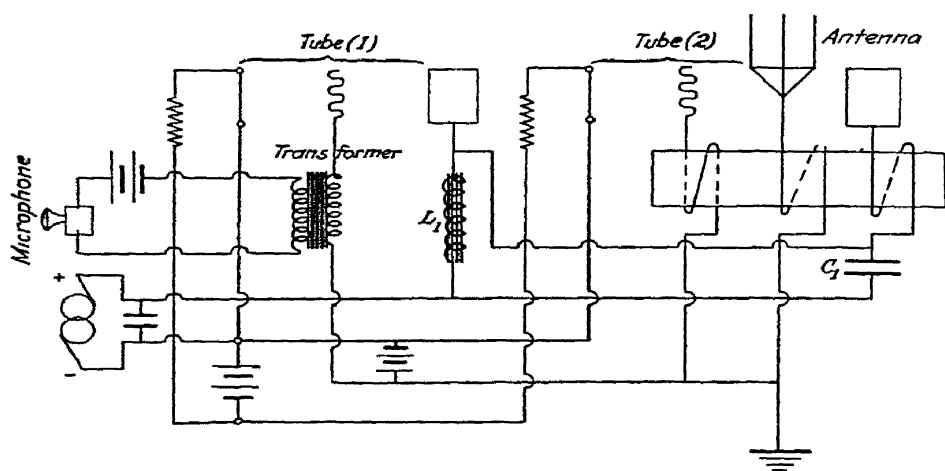



Fig. 69.—Plate circuit modulator.

A typical transmitting circuit in which this system of modulation is utilized is shown schematically by Fig. 69. Tube 1 is the speech amplifier and tube 2 is the high-frequency generator. The voltage due to the voice currents is introduced into the plate circuit of the oscillator by means of the large inductance L_1. The condenser C_1 acts as a high-frequency bypass for the coil L_1.

Problems

17. Figure 70 gives the plate current–grid voltage curve for a C-12 tube for a plate voltage of 20 volts. When operation takes place about the point $E_{gp} = 0$, $E_{pp} = 20$, what is the value of the detecting constant B for this tube? If $G_p = 41 \times 10^{-6}$ mhos for this tube, what are the values of b_1 and b_2 when R_p has the best possible value?

Tube C-12
I_f = 0.25 Amp
E_{pp} = 20 Volts
400
300
200
100
0
Plate Current in Micro Amperes
-4
-3
-2
-1
0
1
Grid Potential Above Negative End of Filament

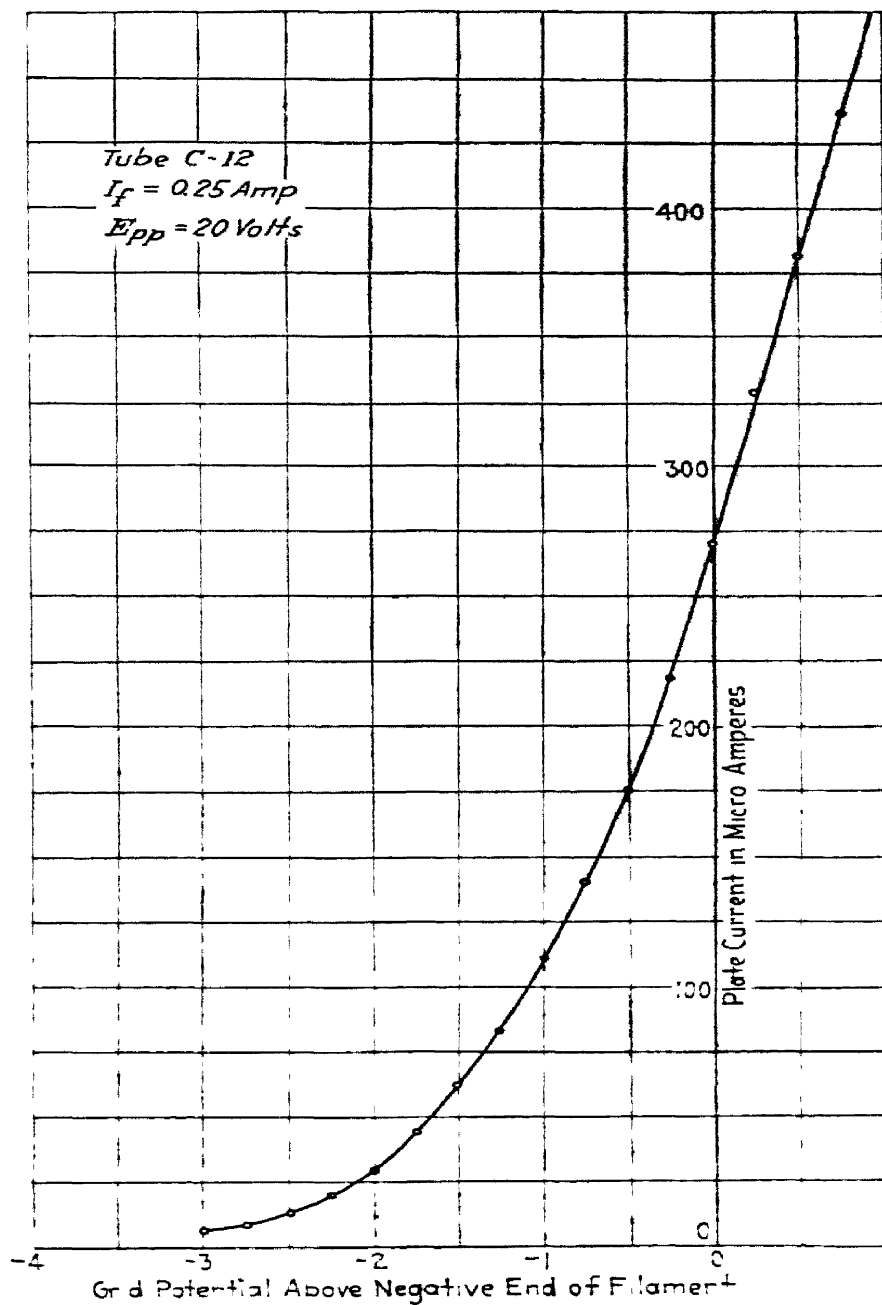



FIG. 70.

CHAPTER VIII

THEORY AND DESIGN OF AMPLIFIERS

53. Internal Capacitance of Triodes.

Little has been said in the preceding chapters concerning the effects of the internal capacitance of vacuum tubes. In some situations these capacitances play but little part in the behavior of the circuits. In other situations, however, the internal capacitances have a marked effect upon the performance of the system. In building up the fundamental methods of analytically treating vacuum tube circuits, the capacitance effects were omitted for the sake of simplicity. It is the purpose in this and the following section to show how the theory and methods already developed must be modified in order to take into account the internal capacitance of the triode.

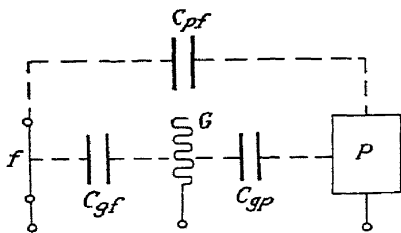



FIG 71 —Internal capacitances of a triode

Strictly speaking, the internal capacitance of a triode is due to the location in close proximity to each other of the filament, plate, and grid. In Fig. 71 the three elements of the triode are shown and the capacitances between them are represented by the condensers which are drawn in with dotted lines. In actual circuits these condensers also should include the capacitance due to the socket and those portions of the lead wires which are at different potentials relative to each other.

Since the individual capacitances cannot be measured separately, to obtain their values we proceed as follows: The grid and plate are tied together, and the capacitance is then measured from the plate and grid to filament. The capacitance measured is

$$C_1 = C_{gf} + C_{pf} \tag{1}$$

The jumper is now removed from the grid and plate, the grid and filament are tied together, and the capacitance is measured from the plate to the filament and grid. This measurement gives

$$C_2 = C_{gp} + C_{pf} \tag{2}$$

In the last measurement the jumper is placed from the filament to the plate, and the capacitance is measured from the grid to the filament and plate. This measurement gives

$$C_3 = C_{gf} + C_{gp} \tag{3}$$

If these three equations are solved for the various capacitances there results

$$C_{gf} = \frac{C_1 + C_3 - C_2}{2} \tag{4}$$

$$C_{gp} = \frac{C_2 + C_3 - C_1}{2} \tag{5}$$

$$C_{pf} = \frac{C_1 + C_2 - C_3}{2} \tag{6}$$

For a 201-A tube with its socket and very short leads these capacitances have the following values.

$$C_1 = 9.9 \times 10^{-12} \text{ farads}$$
$$C_2 = 12.65 \times 10^{-12} \text{ farads}$$
$$C_3 = 14.3 \times 10^{-12} \text{ farads}$$

If these values are substituted in Eqs. (4), (5), and (6) there results

$$C_{gf} = 5.77 \times 10^{-12} \text{ farads}$$
$$C_{gp} = 8.52 \times 10^{-12} \text{ farads}$$
$$C_{pf} = 4.12 \times 10^{-12} \text{ farads}$$

54. Input Impedance of a Triode with Various Impedances in the Plate Circuit.

The circuit to be discussed in this section is shown by Fig. 72. The fundamental equations for this system were developed in Sec. 47. The grid is assumed to be operated with a negative bias so that there is no grid conduction

current. The impedance which would be measured across the terminals *ab* is the input impedance of the triode when used as an amplifier. This input impedance is given by

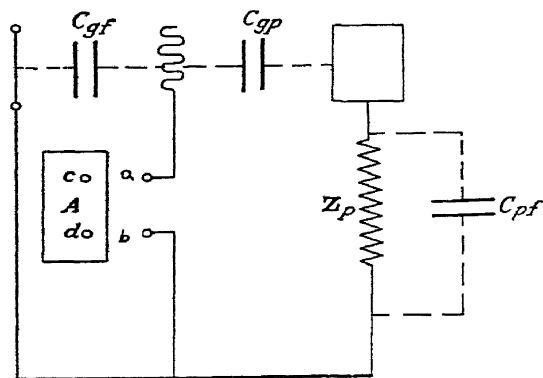



FIG. 72.—Triode amplifier with impedance in the output circuit.

Eq. (64) of Sec. 47. Upon comparing Fig. 72 with Fig. 64, we write

$$\mathbf{Z}_2 = -\frac{j}{\omega C_{gp}} = -jX_{c2} \tag{7}$$

$$\mathbf{Z}_4 = -\frac{j}{\omega C_{gf}} = -jX_{c4} \tag{8}$$

$$\mathbf{Z}_3 = \frac{1}{\frac{1}{\mathbf{Z}_p} + j\omega C_{pf}} = \frac{\mathbf{Z}_p}{1 + j\mathbf{Z}_p\omega C_{pf}} \tag{9}$$

and the equation for the input impedance becomes

$$\mathbf{Z}_{ab} = \frac{-jX_{c4}[-jX_{c2} + \mathbf{Z}_3 - jX_{c2}\mathbf{Z}_3G_p]}{-jX_{c4}[1 + \mathbf{Z}_3(G_p + G_{cp})] - jX_{c2} + \mathbf{Z}_3 - jX_{c2}\mathbf{Z}_3G_p}$$
$$= R_{ab} + jX_{ab} \tag{10}$$

Case I. Pure Resistance in Plate Circuit.—In order to obtain an idea of the manner in which the input impedance varies as the magnitude of a pure resistance load in the plate branch is changed, the values in Table II were calculated. An examination of this table brings out the interesting fact that the resistance component of the input impedance for any one value of output resistance is practically the same for all frequencies up to 1,000,000 cycles per second. The

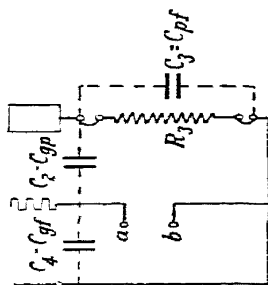
TABLE II.—VARIATION OF INPUT WITH OUTPUT RESISTANCE

R_3	Values of input impedance Z^a_b			
	$f = 1{,}000 \sim$	$f = 10{,}000 \sim$	$f = 100{,}000 \sim$	$f = 1{,}000{,}000 \sim$
0	$0 - j1.115 \times 10^7$	$0 - j1\ 115 \times 10^6$	$0 - j1\ 115 \times 10^5$	$0 - j1\ 115 \times 10^4$
100	$36\ 1 - j1\ 046 \times 10^7$	$36\ 1 - j1\ 046 \times 10^6$	$36\ 1 - j1\ 046 \times 10^5$	$36\ 1 - j1\ 046 \times 10^4$
1,000	$310 - j7\ 10 \times 10^6$	$310 - j7.10 \times 10^5$	$310 - j7\ 10 \times 10^4$	$310 - j7.10 \times 10^3$
10,000	$860 - j2\ 92 \times 10^6$	$860 - j2\ 92 \times 10^5$	$860 - j2\ 92 \times 10^4$	$876 - j3.07 \times 10^3$
100,000	$1{,}027 - j1\ 98 \times 10^6$	$1{,}027 - j1\ 98 \times 10^5$	$1{,}000 - j1.96 \times 10^4$	$1{,}013 - j2.033 \times 10^3$

FIG 73

$$Z_{ab} = \frac{-jX_{c4}[-jX_{c2} + Z_3 - jX_{c2}Z_3G_p]}{-jX_{c4}[1 + Z_3(G_p + G_{cp})] - jX_{c2} + Z_3 - jX_{c2}Z_3G_p}$$

$$Z_3 = \frac{1}{\frac{1}{R_3} + j\omega C_3} = \frac{R_3(1 - j\omega C_3 R_3)}{1 + \omega^2 C_3^2 R_3^2}$$

$C_2 = C_{gp} = 8\ 52 \times 10^{-12}$ farads
$C_3 = C_{pf} = 4\ 12 \times 10^{-12}$ farads
$C_4 = C_{gf} = 5\ 77 \times 10^{-12}$ farads
$G_{gp} = 1\ 08 \times 10^{-8}$ mhos
$G_p = 1\ 27 \times 10^{-4}$

other component of the input impedance is a capacity reactance which, for any one value of output resistance, varies almost inversely with the frequency. That this should be true can be seen from the equations if we proceed as follows: Put $Z_p = R_p$ in Eq. (9). Then for frequencies below 10^5 cycles per second, Z_3 is given very closely by the expression

$$R_3 + jX_3 = R_p - j\omega C_3 R_p^2 \tag{11}$$

Upon substituting Eq. (11) in Eq. (10), rationalizing and solving for R_{ab} and X_{ab}, there results

$$R_{ab} = \frac{\left(\frac{a}{f^2} + b\right)d + \left(\frac{e}{f}\right)\left(\frac{h}{f} - gf\right)}{d^2 + \left(\frac{h}{f} - gf\right)^2} \tag{11a}$$

$$X_{ab} = \frac{\left(\frac{a}{f^2} + b\right)\left(\frac{h}{f} - gf\right) - \left(\frac{e}{f}\right)d}{d^2 + \left(\frac{h}{f} - gf\right)^2} \tag{12}$$

where

$$\frac{a}{f^2} = -X_{c2}X_{c4} - X_{c2}X_{c4}G_pR_p \tag{13}$$

$$b = X_{c4}X_3 = -\frac{C_3}{C_4}R_p^2 \tag{14}$$

$$d = X_{c4}X_3(G_p + G_{cp}) + R_p + X_3X_{c2}G_p$$

$$= b(G_p + G_{cp}) + R_p - \frac{C_3}{C_2}R_p^2G_p \tag{15}$$

$$\frac{e}{f} = X_{c4}R_p + X_{c2}X_{c4}X_3G_p = X_{c4}R_p + bX_{c2}G_p \tag{16}$$

$$\frac{h}{f} = X_{c4} + X_{c4}R_p(G_p + G_{cp}) + X_{c2} + X_{c2}R_pG_p \tag{17}$$

$$gf = X_3 = -\omega C_3 R_p^2 \tag{18}$$

f represents the frequency in cycles per second.

For the tube which we have been considering,

$$a = -11.696 \times 10^{20}$$
$$b = 7.14 \times 10^7$$
$$d = -8.232 \times 10^4$$

$$e = 1.064 \times 10^{14}$$
$$h = 4.029 \times 10^{11}$$
$$g = -2.587 \times 10^{-3}$$

For frequencies below 100,000 cycles per second $\frac{a}{f^2}$ is large compared to b, $\frac{h}{f}$ is large compared to gf, and d^2 is very small compared to $\left(\frac{h}{f}\right)^2$. For frequencies below 100,000 the resistance component of the input impedance of a triode with a pure resistance in the output circuit becomes

$$R_{ab} = \frac{ad - eh}{h^2} \qquad (19)$$

In the expression for the reactance term, $\left(\frac{a}{f^2}\right)\left(\frac{h}{f}\right)$ is large compared to $\left(\frac{e}{f}\right)d$ at the lower frequencies and the expression for the reactance reduces to

$$X_{ab} = \frac{1}{f}\left(\frac{a}{h}\right) \qquad (20)$$

From Eqs. (19) and (20) it is evident that, at the lower frequencies, the input resistance remains constant with the frequency while the input reactance varies inversely with the frequency.

Case II. Inductive Reactance in Plate Circuit.—Table III gives the values of the input impedance when the plate circuit contains an inductive reactance and a resistance. The impedance in the plate circuit was taken as $k(1 + j20)$, and values of input impedance are given for various values of k at several different frequencies. This table brings out the interesting fact that at 1,000 and at 10,000 cycles the resistance component of the input impedance is negative for all values of k up to 100,000. At the higher frequencies the resistance component of the input impedance becomes positive for the larger values of k. The reactive component of the input impedance is a capacity reactance for all of the conditions considered.

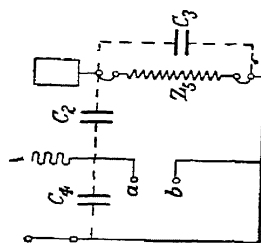
TABLE III.—VARIATION OF INPUT IMPEDANCE WITH OUTPUT IMPEDANCE

	Values of input impedance Z_{ab}			
k	$f = 10^3 \sim$	$f = 10^4 \sim$	$f = 10^5 \sim$	$f = 10^6 \sim$
0	$0 - j1\ 115 \times 10^{7}$	$0 - j1\ 115 \times 10^{6}$	$0 - j1.115 \times 10^{5}$	$0 - j1\ 115 \times 10^{4}$
10^2	$-3\ 98 \times 10^{6} - j4.31 \times 10^{6}$	$-4\ 05 \times 10^{5} - j4\ 68 \times 10^{5}$	$-4\ 03 \times 10^{4} - j4\ 64 \times 10^{4}$	$-3.54 \times 10^{1} - j4\ 11 \times 10^{3}$
10^3	$-5\ 98 \times 10^{5} - j1\ 91 \times 10^{6}$	$-5\ 94 \times 10^{4} - j1\ 91 \times 10^{5}$	$-4.96 \times 10^{3} - j1.90 \times 10^{4}$	$+4.32 \times 10^{2} - j1\ 87 \times 10^{3}$
5×10^3	$-1\ 16 \times 10^{5} - j1\ 85 \times 10^{6}$	$-1\ 10 \times 10^{4} - j1\ 84 \times 10^{5}$	$-1.62 \times 10^{2} - j1.75 \times 10^{4}$	$+9.14 \times 10^{2} - j1\ 89 \times 10^{3}$
10^4	$-6\ 06 \times 10^{4} - j1\ 85 \times 10^{6}$	$-5\ 0 \times 10^{3} - j1\ 84 \times 10^{5}$	$+4\ 22 \times 10^{2} - j1.84 \times 10^{4}$	$+9.65 \times 10^{2} - j1\ 94 \times 10^{3}$
10^5	$-4\ 58 \times 10^{3} - j1\ 84 \times 10^{6}$	$-2\ 05 \times 10^{3} - j1\ 84 \times 10^{5}$	$+9\ 36 \times 10^{2} - j2.07 \times 10^{4}$	$+9.98 \times 10^{2} - j1\ 89 \times 10^{3}$

FIG 74

$$Z_{ab} = \frac{-jX_{c4}[-jX_{c2} + Z_3 - jX_{c2}Z_3G_p]}{-jX_{c4}[1 + Z_3(G_p + G_{op})] - jX_{c2} + Z_3 - jX_{c2}Z_3G_p}$$

$$Z_3 = \frac{Z_5}{1 + jZ_5\omega C_3}$$

$Z_5 = k\ (1 + j20)$
$C_2 = C_{gp} = 8\ 52 \times 10^{-12}$ farads
$C_3 = C_{pf} = 4\ 12 \times 10^{-12}$ farads
$C_4 = C_{gf} = 5\ 77 \times 10^{-12}$ farads
$G_{op} = 1\ 08 \times 10^{-3}$ mhos
$G_p = 1\ 27 \times 10^{-4}$ mhos

When conditions are such that the resistance component of the input impedance is negative, the triode would feed power into a generator connected across the input terminals. Let the box A represent any general impedance which has the terminals c and d connected to the terminals a and b. Let the impedance of the box measured across the terminals cd at any frequency f be $Z_{cd} = R_{cd} + jX_{cd}$. Let the impedance of the tube at this same frequency across the terminals ab be $Z_{ab} = R_{ab} + jX_{ab}$. Then if R_{ab} is negative and greater than R_{cd}, and if $X_{ab} = -X_{cd}$, the system may generate sustained oscillations at the frequency f. This follows from the theory given in Chaps. III and VI. It is thus evident that when the resistance component of the input impedance is negative, it is always possible for the system to generate sustained oscillations if the correct impedance happens to be connected across the input terminals ab. These conditions are very frequently fulfilled in amplifier circuits and lead to a "howling" of the amplifier.

It was stated that when R_{ab} is negative and greater in absolute value than R_{cd} and when $X_{ab} = -X_{cd}$, the system may generate sustained oscillations. We refrain from saying that the system will generate sustained oscillations for the reasons stated below. Let us refer to the theory of the simple series circuit associated with a pure resistance neutralizer as given in Chap. III, Sec. 12. We shall consider the circuit shown by Fig. 26 with the generating device represented by box B replaced by a switch. The condenser C is charged and then the switch is closed. The differential equation for the system then is

$$L\frac{d^2i}{dt^2} + (R - N)\frac{di}{dt} + \frac{i}{C} = 0 \qquad (21)$$

The solution of this equation is

$$i = A_1\epsilon^{g_1t} + A_2\epsilon^{g_2t} \qquad (22)$$

where

$$g_1 = -\frac{R - N}{2L} + \sqrt{\frac{(R - N)^2}{4L^2} - \frac{1}{LC}} \qquad (23)$$

$$g_2 = -\frac{R - N}{2L} - \sqrt{\frac{(R - N)^2}{4L^2} - \frac{1}{LC}} \qquad (24)$$

The solution is oscillatory in character when $\frac{(R - N)^2}{4L^2} < \frac{1}{LC}$. If this condition is fulfilled and if N is greater than R, we have the conditions for sustained oscillations as discussed in Chap. IV. With N greater than R, one other condition may arise, and it is this condition in which we are now interested. Since $R - N$ enters as a square in the conditions for an oscillatory solution of Eq. (21), it is possible to have N greater than R and still have a logarithmic solution of Eq. (21). For the case then in which N is greater than R and $\frac{(R - N)^2}{4L^2} > \frac{1}{LC}$, g_1 and g_2 are positive real numbers, and the current is always increasing in the same direction. The condenser first discharges and then charges up in the reverse direction until it is destroyed or until reactions set up in the neutralizer cause such a lowering in the value of N that the process is stopped.

While this theory may not apply strictly to the case under discussion because of the fact that the box A may contain any arrangement of circuit elements such as inductances, condensers, resistances, and transformers, still as R_{ab} may be much larger in absolute value than R_{cd} (see Table III), instead of generating sustained oscillations, a unidirectional change of potentials on the grid and plate may take place. This change of potentials on the plate and grid may cause the tube to become inoperative or to "choke up."

Case III. Capacity Reactance in Plate Circuit.—Table IV gives the variation of input impedance with output impedance when the output impedance is made up of resistance and capacity reactance. The tube constants used in calculating this table are the same as those used before. The output impedance was taken as $k(1 - j100)$. For this case the values of input resistance are always positive, and the phase angle of the input impedance is much smaller than for the case where there is a pure resistance in the output circuit. This means that more power will have to be expended on the grid in order to maintain a given potential between the grid and filament than would be required for a pure resistance in the output circuit.

TABLE IV.—VARIATION OF INPUT IMPEDANCE WITH OUTPUT IMPEDANCE

k	Values of input impedance Z_{ab}			
	$f = 10^3 \sim$	$f = 10^4 \sim$	$f = 10^5 \sim$	$f = 10^6 \sim$
0	$0 - j1.115 \times 10^7$	$0 - j1.115 \times 10^6$	$0 - j1.115 \times 10^5$	$0 - j1.115 \times 10^4$
10	$4.45 \times 10^6 - j7.65 \times 10^6$	$4.46 \times 10^5 - j7.67 \times 10^5$	$4 \times 10^4 - j2.15 \times 10^4$	$4.56 \times 10^3 - j8.08 \times 10^3$
100	$1.19 \times 10^6 - j2.015 \times 10^6$	$1.19 \times 10^5 - j2.00 \times 10^5$	$1.92 \times 10^4 - j2.15 \times 10^4$	$2.16 \times 10^3 - j2.205 \times 10^3$
500	$2.42 \times 10^5 - j1.845 \times 10^6$	$2.52 \times 10^4 - j1.85 \times 10^5$	$3.11 \times 10^3 - j1.85 \times 10^4$	$1.26 \times 10^3 - j1.94 \times 10^3$
1,000	$1.22 \times 10^5 - j1.84 \times 10^6$	$1.31 \times 10^4 - j1.84 \times 10^5$	$2.21 \times 10^3 - j1.84 \times 10^4$	$1.15 \times 10^3 - j1.92 \times 10^3$
10,000	$1.31 \times 10^4 - j1.84 \times 10^6$	$2.25 \times 10^3 - j1.84 \times 10^5$	$1.39 \times 10^3 - j1.84 \times 10^4$	$1.04 \times 10^3 - j1.90 \times 10^3$

Data and diagram are the same as given with Table III. Output impedance $Z_0 = k(1 - j100)$.

55. Value of the Resistance Which Must Be Placed in the Output Circuit in Order to Obtain Maximum Power Output.

The circuit to be discussed in this section is shown by Fig. 75. The triode has an alternator of negligible impedance in the grid branch and a pure resistance in the plate branch. The grid is supposed to have enough negative bias on it so that it does not become positive during operation. That is, the negative steady potential placed on the grid is greater than the peak voltage of the generator in the grid circuit. The input impedance characteristics of this circuit were discussed in the preceding section. In this section we propose to discuss the power output characteristics of the circuit.

FIG. 75.—Triode amplifier with resistance in output circuit.

The current through the impedance consisting of R_p and C_3 in parallel is given by Eq. (67) of Sec. 47. In this equation

$$Z_2 = -\frac{j}{\omega C_2} = -jX_{c2} \tag{25}$$

$$Z_3 = \frac{1}{\dfrac{1}{R_p} - \dfrac{1}{jX_{c3}}} = \frac{R_p X_{c3}(X_{c3} - jR_p)}{R_p^2 + X_{c3}^2} = R_3 + jX_3 \tag{26}$$

From Eq. (68) of Sec. 47 the power output of the triode is

$$P_0 = I_3^2 R_3 \tag{27}$$

This is the power expended in the utilization resistance R_p. If we try to find the value of R_p which makes this power output a maximum, we are led to a complicated sixth-degree equation. This equation, which gives the best value of R_p, is

$$R_p^6(X_{c3}^2X_{c2}^2G_p^2 + X_{c2}^2 + X_{c3}^2 + 2X_{c2}X_{c3}) + R_p^4(2X_{c3}^4 + 2X_{c2}^2X_{c3}^4G_p^2 + X_{c2}^2X_{c3}^2 + 4X_{c2}X_{c3}^3) - R_p^2(X_{c2}^2X_{c3}^4 - X_{c3}^6 - 2X_{c2}X_{c3}^5 - X_{c2}^2X_{c3}^6G_p^2) - X_{c2}^2X_{c3}^6 = 0 \tag{28}$$

If we divide Eq. (28) by X_{c3}^6 and then let X_{c3} approach infinity, we find that when $X_{c3} = \infty$, R_p should have the value given below for maximum power output.

$$R_p = \sqrt{\frac{X_{c2}^2}{1 + X_{c2}^2 G_p^2}} \quad (X_{c3} = \infty) \tag{29}$$

If $\dfrac{1}{X_{c2}^2}$ is small compared to G_p^2, Eq. (29) takes the familiar form

$$R_p = \frac{1}{G_p} \tag{30}$$

If we use the tube constants given under Tables II, III, or IV these equations give the following values for R_p at 1,000,000 cycles per second:

Equation (30) R_p = 7,880 ohms
Equation (29) R_p = 7,254 ohms
Equation (28) R_p = 6,678 ohms

The value given by Eq. (28) is of course the correct one. The value given by Eq. (29) lies about midway between the values given by Eqs. (30) and (28). As the frequency is lowered, the values given by Eqs. (28) and (29) approach the value given by Eq. (30). From this we conclude that at audio frequencies the value given by Eq. (30) is substantially correct. Since the value of R_p for maximum power output is not a critical one, the value of R_p given by Eq. (30) may be used without an appreciable sacrifice in power output even at the lower radio frequencies. If values which are more nearly correct than those given by Eq. (30) are desired, Eq. (29) may be used. It is hardly ever necessary to use Eq. (28) because, as we will show later, the tube gives either very low values of power amplification or even puts out less alternating power than is furnished to it at very high frequencies with a pure resistance in the output circuit. It is, however, interesting to note the following fact concerning Eq. (28). R_p enters the first term to the sixth power; the reactances enter to the second power. R_p enters the second term to the fourth power; the react-

ances enter it to the fourth power. R_p enters the third term to the second power; the reactances enter it to the sixth power. R_p does not appear in the last term; the reactances enter it to the eighth power. R_p is less than $\frac{1}{G_p}$. Therefore if X_{c2} and X_{c3} are of the same order of magnitude or larger than $\frac{1}{G_p}$, the first term may be dropped without appreciable error, thus reducing Eq. (28) to a double quadratic equation. For the tube for which we have been making calculations, this condition is fulfilled up to 10^7 cycles per second. R_p calculated at 1,000,000 cycles per second by dropping the term containing R_p^2 in Eq. (28) is 6,680 ohms. This value is substantially the same as the correct value 6,678 given above. The best value for R_p at 10,000,000 cycles per second is 1,250 ohms. This shows that at frequencies above 10^6 cycles per second, it is not even approximately correct to take R_p equal to $\frac{1}{G_p}$ for maximum power output.

56. Amplification Characteristics of a Single Tube with a Pure Resistance in the Output or Plate Circuit.

The circuit to be discussed in this section is shown by Fig. 75. The triode has a pure resistance R_p in the plate circuit. The grid is operated with a negative bias so that $G_g = 0$. The fundamental equations for this circuit are those developed in Sec. 47. The Z_3 which enters these equations is

$$Z_3 = \frac{1}{\frac{1}{R_p} + j\omega C_3} = \frac{R_p(1 - j\omega C_3 R_p)}{1 + \omega^2 C_3^2 R_p^2} = R_3 - jX_3 \quad (31)$$

The input impedance to the triode is

$$Z_{ab} = \frac{-jX_{c4}[-jX_{c2} + Z_3 - jX_{c2}Z_3G_p]}{-jX_{c4}\,[1 + Z_3(G_p + G_{cp})] - jX_{c2} + Z_3 - jX_{c2}Z_3G_p} = R_{ab} + jX_{ab} \quad (32)$$

Equation (67) of Sec. 47 gives the output current as a function of the circuit constants and the voltage applied

to the grid. Let the bracketed term of this equation be represented by $\mathbf{Z}_5$ so that

$$\mathbf{E} - \mathbf{I}_3\mathbf{Z}_5 = 0 \tag{33}$$

$$\mathbf{Z}_5 = -\frac{-jX_{c2} + \mathbf{Z}_3 - jX_{c2}\mathbf{Z}_3G_p}{1 + jX_{c2}G_{cp}} \tag{34}$$

$$\mathbf{Z}_5^2 = \frac{(X_{c2}X_3G_p - R_3)^2 + (X_{c2} + X_3 + R_3G_pX_{c2})^2}{1 + X_{c2}^2G_{cp}^2} \tag{35}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_5} \tag{36}$$

The output voltage is

$$\mathbf{E}_0 = -\mathbf{I}_3\mathbf{Z}_3 \tag{37}$$

The voltage amplification is

$$A_v = \frac{E_0}{E} \tag{38}$$

The power output is

$$P_0 = \frac{E_0^2}{R_p} \tag{39}$$

The power input is

$$P_i = \frac{E^2}{Z_{ab}^2}R_{ab} \tag{40}$$

The power amplification is

$$A_p = \frac{P_0}{P_i} \tag{41}$$

The above equations give a step-by-step method of calculating the characteristics of the simple amplifier.

In order to bring out more clearly the properties of the simple amplifier with a pure resistance in the output circuit, Table V has been prepared. The constants used in calculating the values for this table are the same as those used before, namely,

$$C_2 = C_{pg} = 8.52 \times 10^{-12} \text{ farads}$$
$$C_3 = C_{pf} = 4.12 \times 10^{-12} \text{ farads}$$
$$C_4 = C_{gf} = 5.77 \times 10^{-12} \text{ farads}$$
$$G_p = 1.27 \times 10^{-4} \text{ mhos}$$
$$G_{cp} = 1.08 \times 10^{-3} \text{ mhos}$$

TABLE V.—AMPLIFIER CHARACTERISTICS. PURE RESISTANCE IN PLATE CIRCUIT

f	R_p	Z_3	Z_{ab}	Z_b^2
10	8,000	$8,000 - j1\ 657 \times 10^{-2}$	$821.2 - j3.1327 \times 10^{8}$	3.4844×10^{6}
10^2	8,000	$8,000 - j1\ 657 \times 10^{-1}$	$821\ 2 - j3\ 1327 \times 10^{7}$	$3\ 4844 \times 10^{6}$
10^3	8,000	$8,000 - j1\ 657$	$821.2 - j3.1327 \times 10^{6}$	$3\ 4844 \times 10^{6}$
10^4	8,000	$8,000 - j1\ 657 \times 10$	$821\ 2 - j3.1327 \times 10^{5}$	$3\ 4844 \times 10^{6}$
10^5	7,500	$7,497\ 2 - j1\ 4556 \times 10^{2}$	$809.57 - j3.2057 \times 10^{4}$	3.2701×10^{6}
10^6	6,500	$6,321.0 - j1\ 0636 \times 10^{3}$	$785.25 - j3\ 4354 \times 10^{3}$	$2\ 9932 \times 10^{6}$
10^7	1,250	$1,131\ 5 - j3\ 6614 \times 10^{2}$	$333.81 - j758.16$	$1\ 4505 \times 10^{6}$

f	$A_v = \frac{E_0}{E}$	$P_0 = \frac{E_0}{R_p}$	$P_i = \frac{E^2R_{ab}}{Z_{ab}^2}$	$A_p = \frac{P_0}{P_i}$
10	4 2857	$2\ 2959 \times 10^{-3}E^2$	$8.3678 \times 10^{-15}E^2$	$2\ 7438 \times 10^{11}$
10^2	4 2857	$2\ 2959 \times 10^{-3}E^2$	$8\ 3678 \times 10^{-13}E^2$	$2\ 7438 \times 10^{9}$
10^3	4.2857	$2\ 2959 \times 10^{-3}E^2$	$8.3678 \times 10^{-11}E^2$	$2\ 7438 \times 10^{7}$
10^4	4 2857	$2\ 2959 \times 10^{-3}E^2$	$8.3678 \times 10^{-9}E^2$	2.7438×10^{5}
10^5	4 1466	$2\ 2926 \times 10^{-3}E^2$	$7.8776 \times 10^{-7}E^2$	$2\ 9103 \times 10^{3}$
10^6	3 7050	$2\ 1118 \times 10^{-3}E^2$	$6.6502 \times 10^{-5}E^3$	3.1755×10
10^7	0 98747	$0\ 78997 \times 10^{-3}E^2$	$4\ 8635 \times 10^{-4}E^2$	$1\ 6240$

The values used for R_p were such as to satisfy the conditions for maximum power output at the various frequencies considered. These values are given in column 2 of the table. Column 4 gives the input impedances of the triode. This column again brings out the fact that the power required to maintain a given voltage on the grid goes up very fast as the frequency increases. Thus the power required to maintain 1 volt on the grid at 10^5 cycles per second is nearly 10^8 times as great as the power required to maintain 1 volt on the grid at a frequency of 10 cycles per second. This fact is brought out also by the column marked P_i which gives the alternating-current power input to the triode.

The column marked A_v gives the voltage amplification of the triode. The voltage amplification decreases only slightly with increasing frequencies for frequencies below 10^6 cycles per second.

The column marked P_0 gives the power output of the tube. The power output also remains fairly constant up to a frequency of 10^6 cycles per second.

It is interesting to note that for frequencies below 10^5 cycles per second, the simple formulas for voltage amplification and power output given in Chap. II give very accurate results. Thus for frequencies below 10^5 cycles per second the voltage amplification and the power output of a single tube with a pure resistance in the output may be calculated very accurately by using the formulas

$$A_v = \frac{R_p G_{cp}}{1 + R_p G_p} \tag{42}$$

$$P_0 = \frac{E^2 G_{cp}^2 R_p}{(1 + R_p G_p)^2} \tag{43}$$

The last column, marked A_p, gives the power amplification. The power amplification decreases rapidly as the frequency is increased because of the fact that the power required to maintain a given voltage on the grid goes up rapidly with the frequency, whereas the power output

remains nearly constant at the lower frequencies and falls off at frequencies in the radio range.

57. General Equations for the Impedance-coupled Amplifier. In this section the equations for the system of Fig. 76 are to be worked out. Z_1, Z_2, and Z_p represent impedances of any kind which will permit the passage of direct current through them. The grids of both tubes are to be operated

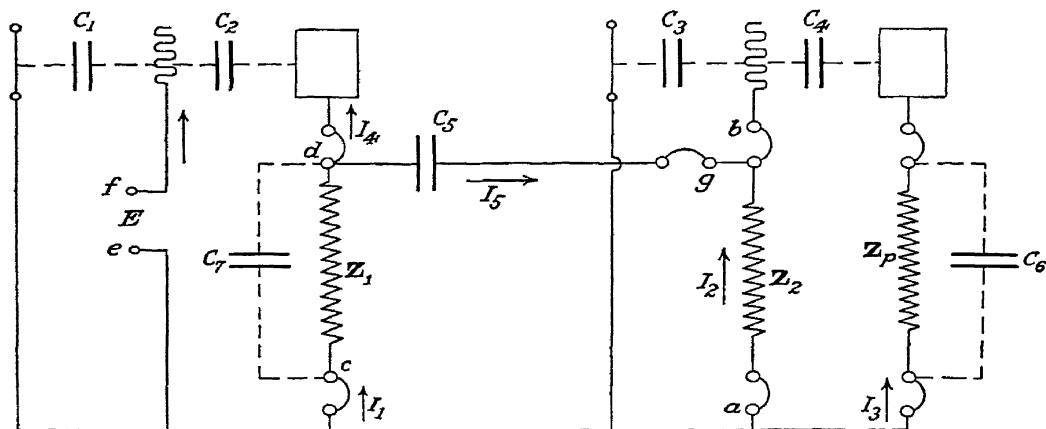



FIG. 76.—Two stage impedance coupled amplifier.

at a potential such that there is no conduction current through the grid to filament space.

The effective output impedance of the second tube is

$$Z_3 = \frac{1}{\frac{1}{Z_p} + j\omega C_6} = \frac{Z_p}{1 + jZ_p\omega C_6} = R_3 - jX_3 \qquad (44)$$

From Eq. (64) of Sec. 47 the impedance from a to b with the links at a and b open is

$$Z_{ab} = \frac{-jX_{c3}[-jX_{c4} + Z_3 - jX_{c4}Z_3G_{p2}]}{-jX_{c3}\,[1+Z_3(G_{p2}+G_{cp2})] - jX_{c4} + Z_3 - jX_{c4}Z_3G_{p2}} = R_{ab} + jX_{ab} \qquad (45)$$

The impedance from g to a with the link at g open and all other links closed is

$$Z_{ag} = \frac{1}{\frac{1}{Z_2} + \frac{1}{Z_{ab}}} \qquad (46)$$

Let

$$\mathbf{Y}_8 = \frac{1}{\mathbf{Z}_1} + j\omega C_7 \tag{47}$$

The impedance from c to d with the links at c and d open and all other links closed is

$$\mathbf{Z}_{cd} = \frac{1}{\mathbf{Y}_8 + \dfrac{1}{\mathbf{Z}_{ag} - jX_{c5}}} \tag{47a}$$

From Eq. (64) of Sec. 47, the input impedance of the amplifier is

$$\mathbf{Z}_{ef} = \frac{-jX_{c1}[-jX_{c2} + \mathbf{Z}_{cd} - jX_{c2}\mathbf{Z}_{cd}G_{p1}]}{-jX_{c1}[1 + \mathbf{Z}_{cd}(G_{p1} + G_{cp1})] - jX_{c2} + \mathbf{Z}_{cd} - jX_{c2}\mathbf{Z}_{cd}G_{p1}} = R_{ef} + jX_{ef} \tag{48}$$

If we make use of Eq. (67) of Sec. 47 we can write for the current $\mathbf{I}_4$

$$\mathbf{I}_4 = -\frac{E(1 + jX_{c2}G_{cp1})}{-jX_{c2} + \mathbf{Z}_{cd} - jX_{c2}\mathbf{Z}_{cd}G_{p1}} \tag{49}$$

The alternating voltage on the plate of the first tube is

$$\mathbf{E}_{p1} = -\mathbf{Z}_{cd}\mathbf{I}_4 \tag{50}$$

The current $\mathbf{I}_5$ is

$$\mathbf{I}_5 = \frac{\mathbf{E}_{p1}}{\mathbf{Z}_{ag} - jX_{c5}} \tag{51}$$

The voltage impressed on the grid of the second tube is

$$\mathbf{E}_{g2} = \mathbf{Z}_{ag}\mathbf{I}_5 \tag{52}$$

From Eq. (67) of Sec. 47 the output current $\mathbf{I}_3$ is

$$\mathbf{I}_3 = -\frac{\mathbf{E}_{g2}(1 + jX_{c4}G_{cp2})}{-jX_{c4} + \mathbf{Z}_3 - jX_{c4}Z_3G_{p2}} \tag{53}$$

The alternating plate voltage on the second tube is

$$\mathbf{E}_{p2} = -\mathbf{Z}_3\mathbf{I}_3 \tag{54}$$

The power output of the amplifier is

$$P_0 = I_3^2R_3 = \frac{E_{p2}^2}{Z_p^2}R_p \tag{55}$$

The power input is

$$P_i = \frac{E^2}{Z_{ef}^2} R_{ef} \tag{56}$$

The power amplification is

$$A_p = \frac{P_0}{P_i} \tag{57}$$

The voltage amplification is

$$A_v = \frac{E_{p2}}{E} \tag{58}$$

Case I. Application to a Resistance-coupled Amplifier.—As an application of these equations let us consider a resistance-coupled amplifier in which the constants have the following values:

$$\begin{aligned}
\mathbf{Z}_1 &= 100{,}000 + j0 \\
\mathbf{Z}_2 &= 500{,}000 + j0 \\
\mathbf{Z}_p &= 10{,}000 + j0 \\
C_1 &= C_3 = C_{gf} = 5.77 \times 10^{-12} \text{ farads} \\
C_2 &= C_4 = C_{gp} = 8.52 \times 10^{-12} \text{ farads} \\
C_6 &= C_7 = C_{pf} = 4.12 \times 10^{-12} \text{ farads} \\
C_5 &= 2.5 \times 10^{-8} \text{ farads} \\
G_{p1} &= G_{p2} = 1.27 \times 10^{-4} \text{ mhos} \\
G_{cp1} &= G_{cp2} = 1.08 \times 10^{-3} \text{ mhos}
\end{aligned}$$

For these constants

$$\mathbf{Z}_{ab} = 857.3 - j2.903\frac{10^9}{f} \tag{59}$$

for values of f up to 10^6 cycles per second.

$$\mathbf{Z}_{ab} = 853.2 - j3{,}047 \text{ at } 10^6 \text{ cycles per second} \tag{60}$$

The amplifications and impedances at various frequencies are given by Table VI. The voltage amplification remains fairly constant for all frequencies between 100 and 100,000 cycles per second and is fairly good even at 10 cycles per second. The power amplification, due to the facts mentioned in the preceding section, falls off rapidly as the frequency increases.

The input impedance to the second tube Z_{ab} drops to 580,000 ohms at a frequency of 5,000 cycles per second (see Eq. (17)). This impedance is in parallel with the resistance in the grid of the second tube Z_2. For this reason Z_2 need not be made much larger than 500,000 ohms.

TABLE VI.—RESISTANCE-COUPLED AMPLIFIER

f	Z_{cd}	Z_{ef}	E_{g2}
10	$9.22 \times 10^4 - j8.28 \times 10^2$	$1.05 \times 10^6 - j1\ 96 \times 10^8$	$4.83E$
100	$8.35 \times 10^4 - j1.945 \times 10^3$	$3.175 \times 10^4 - j1.978 \times 10^7$	$7.685E$
500	$8.35 \times 10^4 - j1.64 \times 10^2$	$6.21 \times 10^3 - j3\ 95 \times 10^6$	$7.69E$
10^3	$8.34 \times 10^4 - j2\ 58 \times 10^3$	$5.00 \times 10^3 - j1.975 \times 10^6$	$7.74E$
2×10^3	$8.31 \times 10^4 - j4\ 84 \times 10^3$	$4.51 \times 10^3 - j9\ 88 \times 10^5$	$7.77E$
3×10^3	$8.27 \times 10^4 - j7.74 \times 10^3$	$5.06 \times 10^3 - j6\ 61 \times 10^5$	$7.77E$
5×10^3	$8.13 \times 10^4 - j1.24 \times 10^4$	$5.00 \times 10^3 - j3\ 91 \times 10^5$	$7.77E$
10^4	$7.6 \times 10^4 - j2\ 35 \times 10^4$	$5.03 \times 10^3 - j1\ 68 \times 10^5$	$7.73E$
10^5	$8.48 \times 10^3 - j2\ 41 \times 10^4$	$5\ 03 \times 10^3 - j2\ 02 \times 10^4$	$7.36E$
10^6	$8\ 06 \times 10^2 - j2\ 79 \times 10^3$	$3.27 \times 10^3 - j4\ 19 \times 10^3$	$2.44E$

f	E_{p2}	P_0	P_i	$A_p = \frac{P_0}{P_i}$
10	$23\ E$	$5.29 \times 10^{-2}E^2$	$2.73 \times 10^{-11}E^2$	1.94×10^9
100	$36.6E$	$0.134E^2$	$8.11 \times 10^{-11}E^2$	$1\ 65 \times 10^9$
500	$36.6E$	$0.134E^2$	$3.98 \times 10^{-10}E^2$	$3\ 37 \times 10^8$
10^3	$36.8E$	$0.1355E^2$	$1\ 28 \times 10^{-9}\ E^2$	$1\ 06 \times 10^8$
2×10^3	$37\ 0E$	$0.137E^2$	$4.62 \times 10^{-9}\ E^2$	$2\ 96 \times 10^7$
3×10^3	$37\ 0E$	$0.137E^2$	$1.16 \times 10^{-8}\ E^2$	$1\ 19 \times 10^7$
5×10^3	$37\ 0E$	$0.137E^2$	$3\ 2 \times 10^{-8}\ E^2$	4.29×10^6
10^4	$36\ 8E$	$0.136E^2$	$1\ 78 \times 10^{-7}\ E^2$	$7\ 61 \times 10^5$
10^5	$35\ 1E$	$0.123E^2$	$1.17 \times 10^{-5}\ E^2$	$1\ 05 \times 10^4$
10^6	$11\ 6E$	$1.34 \times 10^{-2}E^2$	$1\ 16 \times 10^{-4}\ E^2$	$1\ 16 \times 10^2$

In Sec. 54 it was pointed out that capacity reactance in the output circuit of a triode causes the resistance component of the input impedance to increase greatly over the value which it would have with a pure resistance in the output circuit. Now the first tube of the amplifier under consideration has an effective capacity reactance in the output circuit (see column marked Z_{cd}), and therefore the resistance component of the input impedance Z_{ef} has a

larger value than it would have if the output impedance of the first tube were a pure resistance. This fact has an important bearing on the power which must be expended to maintain a given potential on the input of the amplifier.

It is interesting to note that the voltage amplification through the last tube and the power output can be calculated very accurately at all frequencies below 10^5 cycles per second by Eqs. 42 and (43) of Sec. 56.

Case II. Application to an Inductively Coupled Amplifier.—In order to show the application of the equations to an impedance coupled amplifier, the values given in Table VII were calculated. The constants used in making the calculations are given beneath the table. As is evident from the data, the impedance in the plate circuit of the first tube consisted of inductance and resistance in series.

The input impedance to the second tube Z_{ab} has already been calculated and for the range of frequencies under consideration is given by Eq. (60) of this section.

The effective output impedance of the second tube reduces to

$$\mathbf{Z}_3 = 10^4 - j2.589 \times 10^{-3} f \tag{61}$$

The impedance Z_{ag} reduces to

$$\mathbf{Z}_{ag} = \frac{2.147 \times 10^{14} + 4.214\frac{10^{24}}{f^2} - j7.258\frac{10^{20}}{f}}{2.509 \times 10^{11} + 8.427\frac{10^{18}}{f^2}} \tag{62}$$

The effective output impedance of the first tube Z_{cd} is made up of a number of elements in parallel. The reactance component of this impedance changes from inductive reactance to capacity reactance at a frequency lying between 3,000 and 4,000 cycles per second.

The resistance component of the amplifier input impedance Z_{ef} is negative for all frequencies up to 3,000 cycles per second. It takes on positive values for frequencies above 3,000. This could have been predicted from the behavior of Z_{cd}. A triode generally has a negative input

TABLE VII.—IMPEDANCE-COUPLED AMPLIFIER

f	Z_{cd}	Z_{cf}	E_{g2}	E_{p2}	$\frac{P_i}{E^2}$	$\frac{P_0}{E^2}$
10	$605\ 1 + j2{,}001\ 7$	$-3\ 168 \times 10^{8} - j4.846 \times 10^{8}$	$1\ 202E$	$5\ 723E$	-9.462×10^{-10}	$3\ 278 \times 10^{-3}$
50	$1{,}508 + j9{,}992$	$-2{,}228 \times 10^{7} - j4\ 315 \times 10^{7}$	$6\ 044E$	$28\ 8\ E$	$-9\ 455 \times 10^{-9}$	$8\ 295 \times 10^{-2}$
100	$3{,}196 + j1\ 989 \times 10^{4}$	$-9\ 167 \times 10^{6} - j2\ 054 \times 10^{7}$	$6\ 823E$	$32\ 5\ E$	-1.811×10^{-8}	$1\ 056 \times 10^{-1}$
200	$7{,}561 + j3\ 935 \times 10^{4}$	$-1\ 158 \times 10^{6} - j9\ 483 \times 10^{6}$	$8\ 027E$	$38\ 2\ E$	-1.584×10^{-8}	$1\ 462 \times 10^{-1}$
500	$29{,}486 + j9.433 \times 10^{4}$	$-2\ 305 \times 10^{5} - j3\ 745 \times 10^{6}$	$8\ 26\ E$	$39\ 32\ E$	$-1\ 639 \times 10^{-8}$	1.546×10^{-1}
1,000	$9\ 256 \times 10^{4} + j1\ 689 \times 10^{5}$	$-5\ 391 \times 10^{4} - j1\ 865 \times 10^{6}$	$8\ 32\ E$	$39.55\ E$	-1.547×10^{-8}	$1\ 564 \times 10^{-1}$
1,500	$1\ 810 \times 10^{5} + j2\ 123 \times 10^{5}$	$-2\ 110 \times 10^{4} - j1\ 242 \times 10^{6}$	$8.33\ E$	$39\ 61\ E$	-1.367×10^{-8}	1.570×10^{-1}
2,000	$2\ 742 \times 10^{5} + j2\ 171 \times 10^{5}$	$-9\ 926 \times 10^{3} - j9\ 309 \times 10^{5}$	$8\ 34\ E$	$39\ 7\ E$	-1.145×10^{-8}	$1\ 576 \times 10^{-1}$
3,000	$4\ 367 \times 10^{5} + j1\ 078 \times 10^{5}$	$-9\ 311 \times 10^{2} - j4\ 096 \times 10^{5}$	$8\ 35\ E$	$39\ 74\ E$	$-5\ 555 \times 10^{-9}$	$1\ 579 \times 10^{-1}$
4,000	$4\ 789 \times 10^{5} - j5\ 592 \times 10^{4}$	$+1\ 411 \times 10^{3} - j4\ 650 \times 10^{5}$	$8\ 36\ E$	$39\ 79\ E$	$+6\ 672 \times 10^{-9}$	1.583×10^{-1}
5,000	$4\ 083 \times 10^{5} - j1\ 684 \times 10^{5}$	$+2\ 904 \times 10^{3} - j3\ 719 \times 10^{5}$	$8\ 36\ E$	$39.79\ E$	$+2\ 100 \times 10^{-8}$	$1\ 583 \times 10^{-1}$

For diagram, see Fig 76

$Z_1 = 400 + 20f + j200f$

$Z_2 = 5 \times 10^{5} + j0$

$Z_t = 10^{4} + j0$

$C_1 = C_3 = C_{gf} = 5\ 77 \times 10^{-12}$ farads

$C_2 = C_4 = C_{gp} = 8\ 52 \times 10^{-12}$ farads

$C_6 = C_7 = C_{pf} = 4.12 \times 10^{-12}$ farads

$C^5 = 2\ 5 \times 10^{-8}$ farads

$G_{p1} = G_{p2} = 1\ 27 \times 10^{-4}$ mhos

$G_{cp1} = G_{cp2} = 1\ 08 \times 10^{-3}$ mhos

resistance when the output reactance is inductive and a positive input resistance when the output reactance is capacitative. The behavior of a system with negative input resistance has been discussed in Sec. 54.

Because of the negative input resistance of the amplifier the power input as given in column 6 is negative for all frequencies up to 3,000 cycles per second.

The voltages impressed on the grid of the second tube are given in column 4 and the output voltages are given in column 5. These voltages are low at 10 and 50 cycles per second because of the low impedance of Z_1 and the high reactances of C_5 at these frequencies.

The power output is given in column 7. The values of power output are low at 10 and at 50 cycles because of the low voltages impressed on the grid of the second tube at these frequencies.

58. Variation of Voltage Amplification with Output Resistance When a Constant Plate Battery Is Used.

In all of the preceding discussion of amplifiers with a pure resistance in the plate circuit, the conductances G_{cp} and G_p have been taken as constant for all values of output resistance. These conductances will be independent of the output resistance only if the voltage of the plate battery is increased in such a way as to maintain a constant plate potential irrespective of the value of the output resistance. If an amplifier is to be designed to work from a source of fixed plate potential and if a constant grid bias is used, then as the output resistance is increased, the plate potential decreases, due to the fact that the continuous plate space current must flow through the resistance in the plate circuit. It is the purpose of this section to develop a method of arriving at the properties of an amplifier with a pure resistance in the plate circuit when the triode is operated with fixed grid and plate batteries.

The method is best illustrated by means of an example. The characteristic curves for the tube under consideration are given by Fig. 77. The plate battery voltage will be taken as 135 volts and the grid will be operated at -3 volts.

The first curve to be derived is one which gives the relation between resistance in the plate circuit and the

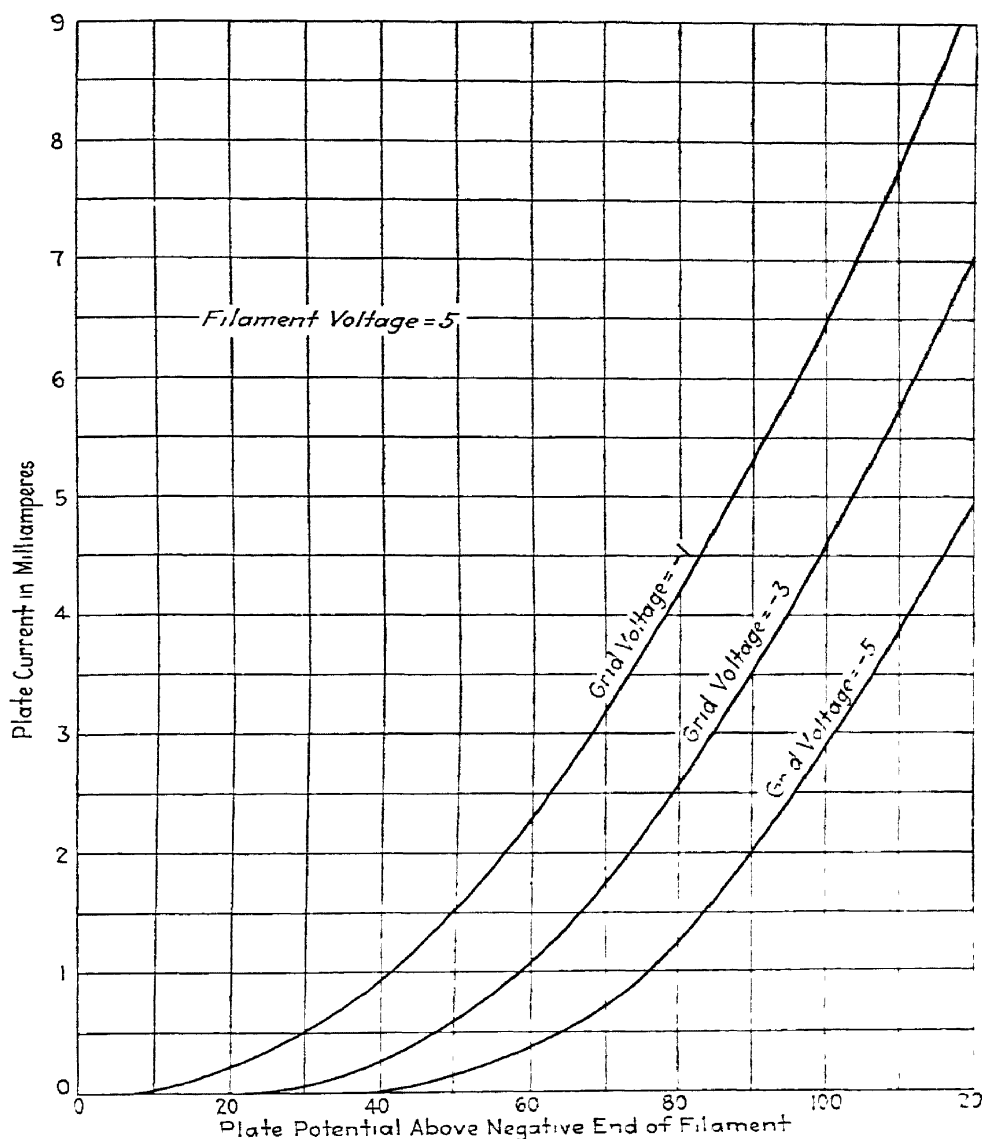



FIG. 77.—Characteristics of $C \times 301$-A tube.

continuous potential impressed on the plate. If the plate voltage is to be 135 volts, the resistance in the plate circuit must be 0. If the plate voltage is to be 70, the plate space

current as read from the curves of Fig. 77 will be 1.7 milliamperes. The drop through the resistance in the plate circuit will be 135 − 70 = 65 volts. Therefore the resistance in the plate circuit which will cause the plate potential to be 70 volts must be $65 \div 1.7 \times 10^{-3} = 38{,}200$ ohms. If the plate potential is to be 60 volts, the plate current will be 1.025 milliamperes. The drop through the resistance

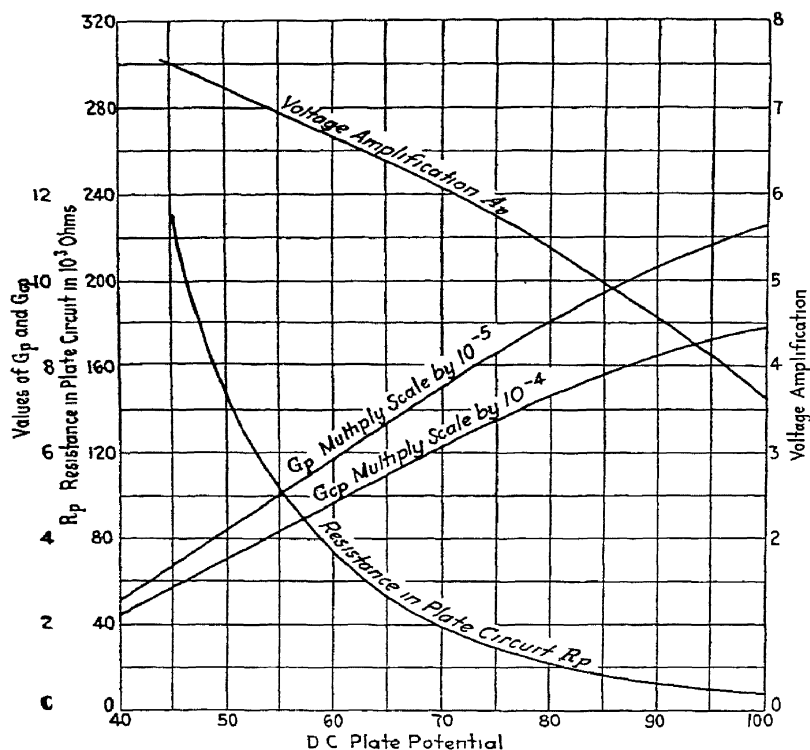



FIG. 78 —Variation of voltage amplification with output resistance when a source of constant plate potential is used

in the plate circuit will be 135 − 60 = 75 volts. Therefore when the resistance in the plate circuit is 75 divided by $1.025 \times 10^{-3} = 73{,}200$ ohms, the plate potential will be 60 volts. By proceeding in this way, the curve of Fig. 78 marked R_p has been derived.

The next curve to be derived is one which shows how the controlled plate conductance varies with the continuous plate potential. This curve is marked G_{cp} on Fig. 78. In

deriving the values for this curve, a peak alternating potential of 2 volts was assumed to be applied to the grid. Thus at 80 plate volts the controlled conductance was found as follows: When the plate voltage is 80 and the grid voltage is -1, the plate current is 4.175×10^{-3} amperes. When the plate potential is 80 and the grid potential is -5, the plate space current is 1.23×10^{-3} amperes. The controlled conductance at 80 volts on the plate then is

$$G_{cp} = \frac{4.175 - 1.23}{4} 10^{-3} = 7.36 \times 10^{-4} \text{ mhos}$$

In deriving the curve which shows the variation of the plate conductance with the continuous plate potential, an estimation has to be made of the peak alternating plate voltage. In deriving the curve marked G_p of Fig. 78 the following peak alternating plate voltages were assumed.

Continuous Plate Potential	Alternating Plate Potential
100	8
90	8
80	10
70	12
60	14
50	14
40	15

When all calculations are made, the alternating plate potentials assumed should be nearly equal to the product of the grid alternating voltage (2 volts in this case) multiplied by the voltage amplification at the continuous plate potential under consideration.

It has been shown in previous sections that at audio frequencies the voltage amplification of a triode with a pure resistance in the output may be calculated by the simple formula

$$A_v = \frac{G_{cp}}{\frac{1}{R_p} + G_p}$$

When R_p has a value of 38,200 ohms, the plate voltage is 70, $G_{cp} = 6.17 \times 10^{-4}$, $G_p = 7.55 \times 10^{-5}$, and the voltage

amplification is 6.05. The other points on the curve marked A_v were obtained in like manner.

It is to be noted that the voltage amplification rises continually as the resistance in the plate circuit is increased, for the range of resistances considered. This is due to the fact that G_p decreases faster than G_{cp} as the resistance is increased. This will not always be true for all tubes and for that matter not always true even for different grid biases on the same tube.

59. The Transformer-coupled Amplifier.

In the preliminary treatment of the transformer-coupled amplifier given in Chap. II, the internal capacitances of the tubes were ignored and the transformers were assumed

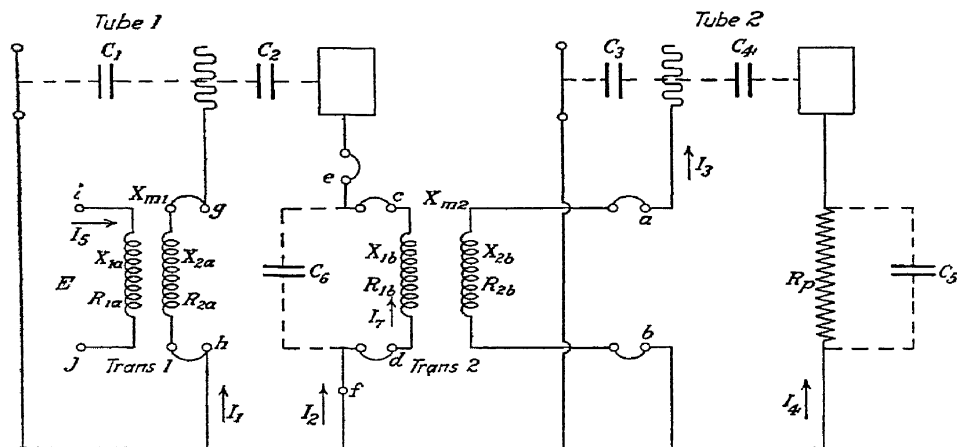



FIG 79 —Two stage transformer coupled amplifier.

to be ideal transformers. In the treatment given in this section we propose to take into account the internal capacitances of the tubes and the fact that the transformers are not ideal ones. The treatment, however, is still only an approximation to the behavior of actual amplifiers because the distributed capacitances of the transformer windings are ignored.

The circuit to be discussed is shown by Fig. 79. The equations for this system are based on the general amplifier equations which were worked out in Sec. 47. The input impedance to the second tube is the impedance which

would be measured across the terminals *ab* with the links at *a* and *b* open. From Eq. (64) of Sec. 47 the expression for this impedance is

$$Z_{ab} = \frac{-jX_{c3}[-jX_{c4}+Z_3-jX_{c4}Z_3G_{p2}]}{-jX_{c3}[1+Z_3(G_{p2}+G_{cp2})]-jX_{c4}+Z_3-jX_{c4}Z_3G_{p2}} \qquad (63)$$

$$= R_{ab} + jX_{ab} \qquad (64)$$

where

$$Z_3 = \frac{1}{\frac{1}{R_p} + j\omega C_5} = \frac{R_p}{1 + jR_p\omega C_5} = R_3 + jX_3 \qquad (65)$$

Equation (63) gives the impedance across the secondary of the second transformer.

The next step is to write the expression for the impedance from terminal *c* to terminal *d* when the links at *c* and *d* are open. If use is made of Eqs. (63) and (64) of Sec. 8, we can write at once

$$Z_{cd} = R_{1b} + \frac{X^2{}_{m2}(R_{ab} + R_{2b})}{(R_{ab} + R_{2b})^2 + (X_{ab} + X_{2b})^2} + j\left[X_{1b} - \frac{X^2_{m2}(X_{ab} + X_{2b})}{(R_{ab} + R_{2b})^2 + (X_{ab} + X_{2b})^2}\right] \qquad (66)$$

$$= R_{cd} + jX_{cd} \qquad (67)$$

The impedance across the terminals *ef* with the link at *e* open is

$$Z_{ef} = \frac{1}{\frac{1}{Z_{cd}} + j\omega C_6} = \frac{Z_{cd}}{1 + jZ_{cd}\omega C_6} = R_{ef} + jX_{ef} \qquad (68)$$

Z_{ef} is the effective output impedance of the first tube. In applying Eq. (64) of Sec. 47 to obtain the input impedance of the first tube, Z_{ef} must be used in place of Z_3. The expression for the impedance across the terminals *gh* with the links at *g* and *h* open is

$$Z_{gh} = \frac{-jX_{c1}[-jX_{c2}+Z_{ef}-jX_{c2}Z_{ef}G_{p1}]}{-jX_{c1}[1+Z_{ef}(G_{p1}+G_{cp1})]-jX_{c2}+Z_{ef}-jX_{c2}Z_{ef}G_{p1}} = R_{gh}+jX_{gh} \qquad (69)$$

The input impedance of the amplifier is

$$\mathbf{Z}_{ij} = R_{ij} + jX_{ij} \tag{70}$$

$$\mathbf{Z}_{ij} = R_{1a} + \frac{X_{m1}^2(R_{gh} + R_{2a})}{(R_{gh} + R_{2a})^2 + (X_{gh} + X_{2a})^2} + j\left[X_{1a} - \frac{X_{m1}^2(X_{gh} + X_{2a})}{(R_{gh} + R_{2a})^2 + (X_{gh} + X_{2a})^2}\right] \tag{71}$$

The current in the primary of the first transformer is

$$\mathbf{I}_5 = \frac{\mathbf{E}}{\mathbf{Z}_{ij}} \tag{72}$$

The current in the secondary of the first transformer is

$$\mathbf{I}_1 = \frac{-jX_{m1}\mathbf{I}_5}{R_{2a} + jX_{2a} + \mathbf{Z}_{gh}} \tag{73}$$

The voltage on the grid of the first tube is

$$\mathbf{E}_{g1} = \mathbf{Z}_{gh}\mathbf{I}_1 \tag{74}$$

From Eq. (67) of Sec. 47 the expression for the current $\mathbf{I}_2$ is

$$\mathbf{I}_2 = -\frac{\mathbf{E}_{g1}(1 + jX_{c2}G_{cp1})}{-jX_{c2} + \mathbf{Z}_{ef} - jX_{c2}\mathbf{Z}_{ef}G_{p1}} \tag{75}$$

The output voltage of the first tube is the voltage across the terminals *cd*. This voltage is given by the equation

$$\mathbf{E}_{dc} = -\mathbf{Z}_{ef}\mathbf{I}_2 \tag{76}$$

The expression for the current in the primary of the second transformer is

$$\mathbf{I}_7 = \frac{\mathbf{E}_{dc}}{\mathbf{Z}_{cd}} \tag{77}$$

The current in the secondary of the second transformer is

$$\mathbf{I}_3 = \frac{-jX_{m2}\mathbf{I}_7}{R_{2b} + jX_{2b} + \mathbf{Z}_{ab}} \tag{78}$$

The voltage on the grid of the second tube is given by the relation

$$\mathbf{E}_{g2} = \mathbf{I}_3\mathbf{Z}_{ab} \tag{79}$$

If we again make use of Eq. (67) of Sec. 47, we can write at once the expression for the output current of the second tube. The expression is

$$\mathbf{I}_4 = -\frac{\mathbf{E}_{g2}(1 + jX_{c4}G_{cp2})}{-jX_{c4} + \mathbf{Z}_3 - jX_{c4}\mathbf{Z}_3G_{p2}} \tag{80}$$

The output voltage of the amplifier is given by the relation

$$\mathbf{E}_0 = -\mathbf{I}_4\mathbf{Z}_3 \tag{81}$$

The voltage amplification through the system is

$$A_v = \frac{E_0}{E} \tag{82}$$

The power output of the second tube is

$$P_0 = I_4^2R_3 \tag{83}$$

The power expended in actuating the system is

$$P_i = I_5^2R_i, \tag{83a}$$

and the expression for the power amplification is

$$A_p = \frac{P_0}{P_i} \tag{84}$$

In order to illustrate the application of these equations to an actual system, the values given in Table VIII have been calculated. The constants used in making the calculations for Table VIII are given under the table. The transformers have a nominal ratio of voltage transformation of 3.5 to 1.

The first column of the table gives the frequencies for which calculations were made. The second column gives values of the input impedance of the amplifier. At frequencies in the vicinity of 1,000 cycles per second, the input resistance of the amplifier is negative.

The third column of Table VIII gives the values of the input impedance to the grid of the first tube. The resistance component of this impedance is negative for all frequencies up to 1,500 cycles per second. If the impedance across the primary of the first transformer has the proper

TABLE VIII.—TRANSFORMER-COUPLED AMPLIFIER

	Z_{if}	Z_{gk}	$\frac{E_{g1}}{E}$	$\frac{E_{do}}{E}$	$\frac{E_{g2}}{E}$	$A_v = \frac{E_{p2}}{E}$
10	$2{,}000 + j8\ 168 \times 10^{2}$	$-8.542 \times 10^{7} - j5\ 283 \times 10^{8}$	1.361	2 524	3 435	16 35
50	$1{,}999 + j4\ 089 \times 10^{3}$	$-3.577 \times 10^{7} - j6.555 \times 10^{7}$	3 283	11.71	37.82	180.1
100	$1{,}984 + j8.197 \times 10^{3}$	$-1\ 267 \times 10^{7} - j2.344 \times 10^{7}$	3.500	19.46	68 06	324 1
200	$1{,}885 + j1\ 665 \times 10^{4}$	$-3{,}500 \times 10^{6} - j9.870 \times 10^{6}$	3.582	26.36	94.27	448 9
500	$850\ 9 + j4\ 770 \times 10^{4}$	$-5.202 \times 10^{5} - j3\ 621 \times 10^{6}$	3 628	30 16	109.0	518 7
1,000	$-1\ 175 \times 10^{4} + j1\ 993 \times 10^{5}$	$-9.176 \times 10^{4} - j1.837 \times 10^{6}$	3 711	31 47	115 4	548 6
1,500	$1{,}014 - j3.149 \times 10^{5}$	$-8.002 \times 10^{3} - j1.223 \times 10^{6}$	3 853	32 76	123 9	589 2
2,000	$7{,}925 - j1\ 013 \times 10^{5}$	$+2.225 \times 10^{4} - j9\ 180 \times 10^{5}$	4 073	34 60	134 4	640 1
3,000	$7{,}889 - j4\ 081 \times 10^{4}$	$+4\ 908 \times 10^{4} - j6\ 141 \times 10^{5}$	4 803	40.52	174 8	831 8
4,000	$8{,}479 - j1\ 965 \times 10^{4}$	$+6\ 768 \times 10^{4} - j4.557 \times 10^{5}$	6 341	52.57	267.9	1,275 3
5,000	$1\ 004 \times 10^{4} - j8\ 948 \times 10^{3}$	$+9\ 284 \times 10^{4} - j3\ 813 \times 10^{5}$	8 282	65.42	434 6	2,068 7

For diagram, see Fig 79

$L_{1a} = L_{1b} = 13$ henrys

$L_{2a} = L_{2b} = 177$ henrys

$M_1 = M_2 = 46\ 8$ henrys

$G_{cp1} = G_{cp2} = 1\ 08 \times 10^{-2}$ mhos

$G_{p1} = G_{p2} = 1\ 27 \times 10^{-4}$ mhos

$C_2 = C_4 = C_{gp} = 8\ 52 \times 10^{-12}$ farads

$C_1 = C_3 = C_{gf} = 5.77 \times 10^{-12}$ farads

$C_5 = C_6 = C_{pf} = 4\ 12 \times 10^{-12}$ farads

$R_{1a} = R_{1b} = 2{,}000$ ohms

$R_{2a} = R_{2b} = 7{,}000$ ohms

$R_p = 10{,}000$ ohms

$Z_{ab} = 857.3 - j2.903 \times 10^{9}$

value, the system will generate sustained oscillations at some frequency below 1,500 cycles per second. Let the impedance across the primary of the first transformer be $R_1 + jX_1$. Then the impedance which would be measured across the secondary of the first transformer with the links at g and h open is

$$Z_{s1} = R_{s1} + jX_{s1} = R_{2a} + \frac{X_{m1}^2(R_{1a} + R_1)}{(R_{1a} + R_1)^2 + (X_{1a} + X_1)^2} + j\left[X_{2a} - \frac{X_{m1}^2(X_{1a} + X_1)}{(R_{1a} + R_1)^2 + (X_{1a} + X_1)^2}\right] \quad (85)$$

If at any frequency below 1,500 cycles per second R_{s1} is less than the absolute value of R_{gh} (R_{gh} is negative) and if $X_{s1} = -X_{gh}$, the system will generate sustained oscillations at this frequency.

Column 4 of Table VIII gives the voltage transformation ratio of the first transformer with the tube across its secondary. Column 5 gives the voltage multiplication through the first transformer and tube. Column 6 gives the voltage multiplication due to the first transformer, the first tube, and the second transformer. Column 7 gives the voltage amplification through the entire system.

It should be recognized that the values of amplification given in the table are for the case in which the voltage is applied to the primary of the first transformer through zero impedance. If the voltage were applied to the first transformer through an impedance, the values of amplification might be quite different from those given in the table. The actual values, however, could be obtained from those given in the table by methods which now should be quite obvious to the reader. Marked deviations will take place from the values given in the table if the impedance across the primary is such as nearly to fulfil the conditions for sustained oscillations at some frequency below 1,500 cycles per second. This value of impedance would result in regenerative amplification, and a peak would occur on the amplification curve at the frequency for which the conditions for sustained oscillations were nearly fulfilled.

It is interesting to note that for frequencies below 1,000 cycles per second, the equations which ignore the internal capacitances of the tubes give very accurate values for the voltage amplification. For frequencies above 1,000 cycles per second, the internal capacitances of the tubes play a very important part in determining the amplification of the system. This statement is not true for all transformers because, as the impedances of the transformer windings are increased, the frequencies at which the internal capacitances of the tubes become important occur at lower and lower values.

The manner in which the impedances Z_i, and Z_{gh} vary as 5,000 cycles is approached suggests that a resonant point lies somewhere immediately above 5,000 cycles per second. This fact is also brought out by the rapidity of the rise in the amplification as a frequency of 5,000 cycles per second is approached.

It is possible to eliminate to a large extent the resonant effects due to the capacitances of the tubes and also those due to the distributed capacitances of the transformer windings by placing a resistance across the secondary of each transformer. For the circuit under discussion, if we are interested in flattening out the amplification curve for frequencies up to 5,000 cycles per second, a resistance of about 250,000 ohms should be placed across the secondary of each transformer. The value of 250,000 was given because the input impedance of the tubes at 5,000 cycles is about 400,000 ohms. The resistance placed across the secondary of each transformer is in parallel with the input impedance of a tube, and if the net impedance across the secondaries of the transformers is to be substantially a pure resistance at all frequencies, the resistance used must be somewhat lower than the input impedance of the tubes at the highest frequency considered. The transformer coupled amplifier with resistance across the secondary of each transformer gives a fairly good quality of amplification and at the same time gives much more amplification per stage than can be obtained with resistance or impedance coupling.

60. The Elimination of the Effects of Grid-to-plate Capacitance.

From the treatment of vacuum tube circuits given in this chapter it is evident that the plate-to-filament and the grid-to-filament capacitances behave the same as capacitances in parallel with the external impedances placed in the plate and grid circuits, respectively. The plate-to-grid capacitance, however, behaves as an inherent feed-back element. The negative input resistance obtained when the output impedance has a net inductive reactance is due to the presence of the plate-to-grid capacitance. When the input resistance is negative, there is always the possibility

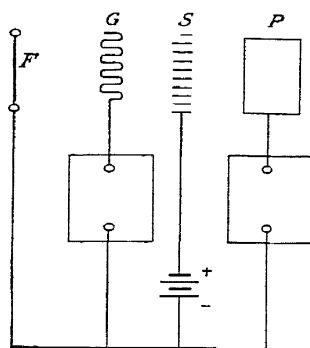



FIG. 80.—Triode with shielded grid.

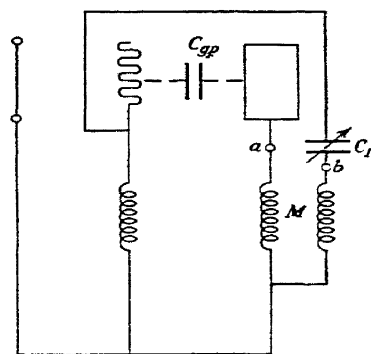



FIG. 81.—Scheme for balancing out the effects of plate to grid capacitance.

that the system will generate sustained oscillations. As a matter of fact, amplifier systems designed to operate at high frequencies are very likely to generate sustained oscillations unless precautions are taken to adjust the system so that oscillations cannot take place.

The most effective way of eliminating the effects of the plate-to-grid capacitance is to shield the grid from the plate in such a way that charges placed on the plate have little or no effect upon the potential of the grid. Such a shielded tube has recently been developed.[1] The shield of this tube consists of thin metal strips placed between the plate and

[1] See HULL, A. W., and N. H. WILLIAMS, "Characteristics of Shielded Pliotrons," *Phys. Rev.*, Vol. 27, No. 4, April, 1926.

grid as shown schematically by Fig. 80. The shield must be maintained at a potential above the common bus as indicated in the diagram. These tubes have not as yet come into commercial use.

The effects of the plate-to-grid capacitance can be balanced out under certain conditions by the following method. Consider the system shown by Fig. 81. In this system let $C_1 = C_{gp}$ and let the adjustment of the coils be such that the potential of the point b above the common bus is at all instants of time equal in magnitude but opposite in sign to the potential of the point a. Under these conditions C_1 neutralizes the effect of C_{gp}. This arrangement is the basis of the neutrodyne method of preventing sustained oscillations in radio frequency amplifiers. The balance need not be exact in order to eliminate oscillations, and C_1 may be used as a control of regenerative amplification.

APPENDIX A

PART I

a. Voltage and Frequency of the Generators Replacing Voltage Induced in a Receiving Antenna by an Interrupted Continuous Wave Transmitter.

The schematic diagram of the electromotive force induced in the antenna in this case is assumed to be as given by Fig. 47. The signal is taken as a series of dots and spaces. The time duration of each space is represented by $2pT$ and of each dot by $2qT$. In order to simplify the calculations, the wave is taken as symmetrical with the origin, and the time intervals pT and qT are assumed to be exactly divisible by the period of the operating frequency. Under these conditions the voltage induced in the antenna is expansible from $t = -\infty$ to $t = +\infty$ in the form

$$e(t) = \sum_{m=1}^{\infty} a_m \sin \frac{m\pi t}{T} \tag{1}$$

where

$$a_m = \frac{2}{T}\int_0^T e(\lambda) \sin \frac{m\pi\lambda}{T} d\lambda \tag{2}$$

But

$$e(t) = 0 \text{ from } t = 0 \text{ to } t = pT \tag{3}$$

$$= E \sin [\omega(t - pT)] \text{ from } t = pT \text{ to } T \tag{4}$$

Upon substituting Eqs. (3) and (4) in Eq. (2), we obtain

$$a_m = \frac{2E}{T}\int_{pT}^{T} \sin \omega[(\lambda - pT)] \sin \frac{m\pi\lambda}{T} d\lambda \tag{5}$$

$$= \frac{E}{T}\int_{pT}^{T} \left\{ \cos \left[\left(\omega - \frac{m\pi}{T}\right)\lambda - \omega pT\right] - \cos \left[\left(\omega + \frac{m\pi}{T}\right)\lambda - \omega pT\right]\right\} d\lambda \tag{6}$$

If $\omega \neq \frac{m\pi}{T}$, this becomes

$$a_m = \frac{E}{T}\left[\frac{\sin [\omega qT - m\pi]}{\omega - \frac{m\pi}{T}} - \frac{\sin [\omega qT + m\pi]}{\omega + \frac{m\pi}{T}}\right] + \frac{E}{T}(\sin mp\pi)\left[\frac{1}{\omega + \frac{m\pi}{T}} + \frac{1}{\omega - \frac{m\pi}{T}}\right] \quad (6a)$$

Because of the conditions on pT and qT the first term is zero, and we therefore arrive at

$$a_m = \frac{E}{T}(\sin mp\pi)\left[\frac{2\omega}{\omega^2 - \frac{m^2\pi^2}{T^2}}\right] \quad (7)$$

for $\omega \neq \frac{m\pi}{T}$

If $\omega = \frac{m\pi}{T}$, Eq. (5) becomes

$$a_{\frac{\omega T}{\pi}} = \frac{E}{T}\int_{pT}^{T}[1 - \cos (2\omega\lambda - \omega pT)]d\lambda$$

$$= Eq - \frac{E}{T}\left[\frac{\sin (2\omega\lambda - \omega pT)}{2\omega}\right]_{pT}^{T}$$

$$= Eq - \frac{E}{2\omega T}[\sin [\omega T(2 - p)] - \sin \omega pT]$$

Both sine terms vanish in the last equation because of the conditions on p, q, and T. We therefore have

$$a_{\frac{\omega T}{\pi}} = Eq \quad (8)$$

b. The Approximation in Deriving Eq. (48) from Eq. (46) of Sec. 31.

The approximation involved in this step entered because $1 + \frac{n}{4f_0T}$ was taken equal to unity. At a transmitting speed of 30 words per minute $T = 0.05$ second. If $f_0 = 10^6$ cycles per second, the value of the term dropped is equal to $(n)5 \times 10^{-6}$. That is, the term for which $n = 2{,}000$ is in error by only 1 per cent; all terms for which n

is less are in error by less than 1 per cent. If $f_0 = 20{,}000$ (wave length 15,000 meters), the value of the term dropped is $2.5n \times 10^{-4}$ and n may have a value of 40 before the error reaches 1 per cent. At 150 words per minute the value of n for which the error becomes 1 per cent is one-fifth of that given above, namely 400 at a frequency of 1,000,000 cycles per second and 8 at a frequency of 20,000 cycles per second.

PART II

c. Voltage and Frequency of the Generators Replacing the Voltage Induced in a Receiving Antenna by a Spark Station.

For the purposes of this discussion the voltage induced in a receiving antenna is taken to have the wave form shown schematically by Fig. 49. The voltage is assumed to have a peak value equal to E, a frequency $f_0 = \frac{\omega}{2\pi}$, and a logarithmic decrement equal to $\frac{\alpha}{f_0}$. The decrement is large enough so that the voltage due to one spark falls substantially to zero before the next spark takes place. Under these conditions the voltage can be represented as a function of time from $t = -\infty$ to $t = +\infty$ by

$$e(t) = \sum_{m=-\infty}^{m=+\infty} A_m \epsilon^{j\frac{m\pi t}{T}} \tag{9}$$

where

$$A_m = \frac{1}{2T}\int_{-T}^{T} e(\lambda)\epsilon^{-j\frac{m\pi\lambda}{T}}\,d\lambda \tag{10}$$

The expansion is used in the above form to facilitate the evaluation of the Eq. (10). As we have taken the wave form

$$e(t) = E\epsilon^{-\alpha(t+T)} \sin\ [\omega(t + T)]\Big|_{t=-t}^{t=0} \tag{11}$$

$$= E\epsilon^{-\alpha t} \sin \omega t\Big|_{t=0}^{t=T} \tag{12}$$

Substituting these values in Eq. (10), there results

$$A_m = \frac{E}{2T}\left\{\int_{-T}^{0} \epsilon^{-\alpha T}\epsilon^{-\left(\alpha+\frac{jm\pi}{T}\right)\lambda} \sin\left[\omega(\lambda + T)\right]d\lambda + \int_{0}^{T} \epsilon^{-\left(\alpha+\frac{jm\pi}{T}\right)\lambda} \sin \omega\lambda d\lambda\right\} \quad (13)$$

In the first integral put $\gamma = \lambda + T$ and in the second put $\gamma = \lambda$. Equation (13) then becomes

$$A_m = \frac{E}{2T}(1 + \epsilon^{jm\pi})\int_0^T \epsilon^{-\left(\alpha+\frac{jm\pi}{T}\right)\gamma} \sin \omega\gamma d\gamma \quad (14)$$

$$= \frac{E}{2T}(1 + \epsilon^{jm\pi})\left[\left(\epsilon^{-\left(\alpha+\frac{jm\pi}{T}\right)\gamma}\right) \frac{-\left(\alpha + \frac{jm\pi}{T}\right)\sin \omega\gamma - \omega \cos \omega\gamma}{\left(\alpha + \frac{jm\pi}{T}\right)^2 + \omega^2}\right]_{\gamma=0}^{\gamma=T} \quad (15)$$

Since $\epsilon^{-\alpha T}$ is nearly zero, Eq. (15) vanishes at the upper limit, and we have

$$A_m = \frac{E}{2T}[1 + \epsilon^{jm\pi}]\frac{\omega}{\left(\alpha + \frac{jm\pi}{T}\right)^2 + \omega^2} \quad (16)$$

$$[1 + \epsilon^{jm\pi}] = [1 + \cos m\pi + j \sin m\pi]$$

$$= 0 \text{ if } m \text{ is odd and } 2 \text{ if } m \text{ is even}$$

$$A_m = \frac{E}{T}\frac{\omega}{\left(\alpha + \frac{jm\pi}{T}\right)^2 + \omega^2} \text{ for } m \text{ even} \quad (17)$$

$$= 0 \text{ for } m \text{ odd} \quad (18)$$

Equation (9) may be written in the form

$$e(t) = A_0 + \sum_{m=1}^{\infty}\left\{(A_{+m} + A_{-m}) \cos \frac{m\pi t}{T} + j(A_{+m} - A_{-m}) \sin \frac{m\pi t}{T}\right\} \quad (19)$$

Now A_{+m} and A_{-m} are conjugate complex numbers.

Let

$$A_{+m} = a_m + jb_m \text{ and } A_{-m} = a_m - jb_m \tag{20}$$

Then Eq. (19) becomes

$$e(t) = A_0 + \sum_{m=1}^{\infty}\left[2a_m \cos\frac{m\pi t}{T} - 2b_m \sin\frac{m\pi t}{T}\right] \tag{21}$$

$$= A_0 + \sum_{m=1}^{\infty} B_m \cos\left(\frac{m\pi t}{T} + \theta_m\right) \tag{22}$$

where

$$B_m = 2\sqrt{a_m^2 + b_m^2} = 2|A|_m \tag{23}$$

$$\theta_m = \tan^{-1}\frac{b_m}{a_m} \tag{24}$$

From Eqs. (17), (18), and (23) we write

$$B_m = 0 \text{ if } m \text{ is odd} \tag{25}$$

$$= \frac{2E}{T}\frac{\omega}{\sqrt{\left[\alpha^2 + \omega^2 - \frac{m^2\pi^2}{T^2}\right]^2 + \frac{4\alpha^2 m^2\pi^2}{T^2}}} \tag{26}$$

$$= \frac{2E}{\omega T}\frac{1}{\sqrt{\frac{\alpha^4}{\omega^4} + 2\left[1 + \left(\frac{m\pi}{\omega T}\right)^2\right]\frac{\alpha^2}{\omega^2} + \left[1 - \left(\frac{m\pi}{\omega T}\right)^2\right]^2}} \tag{27}$$

In any spark transmitted $\frac{\alpha^4}{\omega^4}$ is very small compared to the other terms under the radical and may therefore be dropped without appreciable error. The value of m which gives the operating frequency is $m = 2f_oT$, if f_oT is an integer. Take f_oT as an integer, and since m must be a positive even integer, take

$$m = 2f_oT + 2n \text{ and } n = 0, \pm 1, \pm 2, \text{ etc.} \tag{28}$$

From Eqs. (27) and (28), we find that the generator whose frequency differs from the operating frequency by $\frac{n}{T}$ cycles per second has a voltage given by

$$B_n = \frac{2E}{\omega T}\frac{1}{\sqrt{2\left[2 + \frac{2n}{f_oT} + \frac{n^2}{f_o^2T^2}\right]\frac{\alpha^2}{\omega^2} + \left[\frac{2n}{f_oT} + \frac{n^2}{f_o^2T^2}\right]^2}} \tag{29}$$

For the poorest transmitting stations (log dec = 0.2), $\frac{\alpha^2}{\omega^2}$ does not exceed 10^{-3}. Therefore the term under the radical of Eq. (29) has the form $2[2 + A]10^{-3} + A^2$. If A is small compared to 2, the A in the first term may be dropped. If A is of the same order of magnitude as 2 or greater, the whole first term may be dropped. Therefore in all cases B_n is given very closely by

$$B_n = \frac{2E}{\omega T} \frac{1}{\sqrt{\frac{4\alpha^2}{\omega^2} + \left[\frac{2n}{f_o T} + \frac{n^2}{f_o^2 T^2}\right]^2}} \tag{30}$$

PART III

d. Voltage and Frequency of the Generators Replacing the Voltage Induced in a Receiving Antenna by Static.

For the purposes of this discussion, the wave form of the voltage induced in a receiving station by static is assumed to be as given by Fig. 51. The time duration of the impulse is $2qT$ seconds, and the time between impulses is $2pT$ seconds.

The voltage as a function of time in the interval $t = -T$ to $t = +T$ is then given by

$$\left.\begin{aligned} e(t) &= -\rho(t + pT)\Big|_{t=-T}^{t=-pT} \\ &= 0\Big|_{t=-pT}^{t=+pT} \\ &= \rho(t - pT)\Big|_{t=pT}^{t=T} \end{aligned}\right\} \tag{31}$$

Since the voltage wave is symmetrical with respect to the voltage axis, it may be represented from $t = -\infty$ to $t = +\infty$ by the cosine series

$$e(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos\frac{m\pi t}{T} \tag{32}$$

$$a_m = \frac{2}{T}\int_0^T e(\lambda) \cos\frac{m\pi\lambda}{T} d\lambda \tag{33}$$

Upon substituting Eq. (31) in Eq. (33), we obtain

$$a_m = \frac{2\rho}{T}\int_{pT}^{T}(\lambda - pT)\cos\frac{m\pi\lambda}{T}\,d\lambda$$

$$= \frac{2\rho T}{m^2\pi^2}(\cos m\pi - \cos mp\pi) \text{ if } m \neq 0 \tag{34}$$

$$a_0 = \frac{2\rho}{T}\int_{pT}^{T}(\lambda - pT)d\lambda = \frac{2\rho}{T}\left[\frac{\lambda^2}{2} - pT\lambda\right]_{pT}^{T}$$

$$= 2pT[\tfrac{1}{2} + \tfrac{1}{2}p^2 - p] \tag{35}$$

APPENDIX B

Solution of the Differential Equations for the Circuit of Figure 25

Summing voltages around circuit 1 gives

$$E - R_1 i_1 - L_1 \frac{di_1}{dt} - \frac{q_1}{C_1} - M_p \frac{di_p}{dt} = 0 \tag{1}$$

$$\frac{dE}{dt} - R_1 \frac{di_1}{dt} - L_1 \frac{d^2 i_1}{dt^2} - \frac{i_1}{C_1} - M_p \frac{d^2 i_p}{dt^2} = 0 \tag{2}$$

The voltage induced in the plate circuit is

$$-M_p \frac{di_1}{dt}$$

The voltage induced in the grid circuit is

$$-M_g \frac{di_1}{dt} \tag{4}$$

The total plate voltage is

$$e_p = -M_p \frac{di_1}{dt} - L_p \frac{di_p}{dt} - R_p i_p \tag{5}$$

The plate current (neglecting the plate capacity) is

$$i_p = \frac{di_1}{dt}\left[-M_g G_{cp} - M_p G_p\right] - L_p G_p \frac{di_p}{dt} - R_p G_p i_p \tag{6}$$

The solution of Eqs. (2) and (6) for the permanent terms has already been obtained by means of the complex algebra. The solution for the transient terms will now be taken up. The two equations to be solved are

$$R_1 \frac{di_1}{dt} + L_1 \frac{d^2 i_1}{dt^2} + \frac{i_1}{C_1} + M_p \frac{d^2 i_p}{dt^2} = 0 \tag{7}$$

$$L_p G_p \frac{di_p}{dt} - \frac{di_1}{dt}\left[-M_g G_{cp} - M_p G_p\right] + D i_p = 0 \tag{8}$$

where

$$D = (1 + R_p G_p) \tag{9}$$

The solution will be of the form

$$i_1 = A_1\epsilon^{gt} \tag{10}$$

$$i_p = A_2\epsilon^{gt} \tag{11}$$

Upon substituting Eqs. (10) and (11) in Eq. (7), there results

$$\left[R_1g + L_1g^2 + \frac{1}{C_1}\right]A_1 + M_pg^2A_2 = 0 \tag{12}$$

Substituting in Eq. (8) gives

$$[L_pG_pg + D]A_2 - g\frac{h}{M_p}A_1 = 0 \tag{13}$$

where

$$h = M_p[-M_gG_{cp} - M_pG_p] \tag{14}$$

Solving Eq. (13) for A_2,

$$A_2 = \frac{ghA_1}{(L_pG_pg + D)M_p} \tag{15}$$

Substituting Eq. (15) in Eq. (12) and collecting terms, there results

$$\left[R_1L_pG_pg^2 + DR_1g + L_1L_pG_pg^3 + DL_1g^2 + \frac{D}{C_1} + \frac{L_pG_pg}{C_p} + g^3h\right]A_1 = 0 \tag{16}$$

Since A_1 cannot be zero, the bracketed term must equal zero and there results

$$(L_1L_pG_p + h)g^3 + (R_1L_pG_p + DL_1)g^2 + \left(DR_1 + \frac{L_pG_p}{C_1}\right)g + \frac{D}{C_1} = 0 \tag{17}$$

For the oscillatory solution the roots of Eq. (17) will be of the form $(a + j\beta)$, $(a - j\beta)$, and g_3 where a, β, and g_3 are real numbers. Setting up an equation with the above terms as roots, we have

$$(g - a - j\beta)(g - a + j\beta)(g - g_3) = 0 \tag{18}$$

or

$$g^3 - (g_3 + 2a)g^2 + (2ag_3 + a^2 + \beta^2)g - g_3(a^2 + \beta^2) = 0 \quad (19)$$

Upon comparing Eq. (19) with Eq. (17), we write

$$-g_3 - 2a = \frac{DL_1 + R_1L_pG_p}{L_1L_pG_p + h} \quad (20)$$

$$a^2 + \beta^2 + 2ag_3 = \frac{R_1DC_1 + L_pG_p}{C_1L_1L_pG_p + C_1h} \quad (21)$$

$$-g_3(a^2 + \beta^2) = \frac{D}{C_1L_1L_pG_p + C_1h} \quad (22)$$

A practical solution of these equations may be obtained by neglecting relatively small terms. In order to obtain an idea of the relative magnitude of the terms in Eqs. (20), (21), and (22), we will consider the data given in Sec. 14. a should not be far from the exponent of the damping factor of circuit 1 alone, and g_3 should not have a value far different from the exponent of the plate circuit alone, so that approximately we have

$$a = \frac{R_1}{2L_1} = \frac{60}{(2)(20{,}320)10^{-6}} = 1{,}475$$

$$g_3 = \frac{1}{L_pG_p} = \frac{10^{12}}{(3{,}350)(200)} = 1.47 \times 10^6$$

Neglecting $2a$ compared to g_3 in Eq. (20), there results

$$g_3 = -\frac{DL_1 + R_1L_pG_p}{L_1L_pG_p + h} \quad (23)$$

Neglecting a^2 compared to β^2 in Eq. (22), there results

$$\beta = \Omega_r\sqrt{\frac{DL_1}{DL_1 + R_1L_pG_p}} \text{ where } \Omega_r = \frac{1}{\sqrt{L_1C_1}} \quad (24)$$

Neglecting a^2 compared to $\beta^2 + 2ag_3$ in Eq. (21), we have

$$a = \frac{\beta^2L_1L_pG_p + \beta^2h - DR_1 - \dfrac{L_pG_p}{C_1}}{2(DL_1 + R_1L_pG_p)} \quad (25)$$

If in the first term of the numerator of Eq. (25) we write

$$\beta^2 = \Omega_r^2 = \frac{1}{L_1 C_1}$$

we have

$$a = -\frac{DR_1 - \beta^2 h}{2(DL_1 + R_1 L_p G_p)} \tag{25a}$$

The equation for the current in the circuit 1 may now be written

$$i_1 = i_s + A_1 \epsilon^{(a+j\beta)t} + A_2 \epsilon^{(a-j\beta)t} + A_3 \epsilon^{g_3 t} \tag{26}$$

And the current in the plate circuit is

$$i_p = i_{ps} + B_1 \epsilon^{(a+j\beta)t} + B_2 \epsilon^{(a-j\beta)t} + B_3 \epsilon^{g_3 t} \tag{26a}$$

In these equations i_s stands for the steady-state terms. Since g_3, the exponent of the damping factor associated with the terms introduced by the plate circuit, is very large compared to a_1, the last term of Eqs. (26) and (26a) will be very rapidly damped out as compared to the other two transient terms of these two equations. The last term will therefore be dropped from Eqs. (26) and (26a). Substituting Eqs. (26) and (26a) in Eq. (2), there results

$$\frac{dE}{dt} - R_1\left[\frac{di_s}{dt} + (a + j\beta)A_1\epsilon^{(a+j\beta)t} + (a - j\beta)\epsilon^{(a-j\beta)t}A_2 + g_3 A_3 \epsilon^{g_3 t}\right]$$
$$- L_1\left[\frac{d^2 i_s}{dt^2} + (a + j\beta)^2 A_1 \epsilon^{(a-j\beta)t} + (a - j\beta)A_2\epsilon^{(a-j\beta)t} + g_3^2 A_3 \epsilon^{g_3 t}\right]$$
$$- \frac{1}{C_1}[i_s + A_1\epsilon^{(a+j\beta)t} + A_2\epsilon^{(a-j\beta)t} + A_3\epsilon^{g_3 t}]$$
$$- M_p\left[\frac{d^2 i_{ps}}{dt^2} + (a + j\beta)^2 B_1\epsilon^{(a-j\beta)t} + (a - j\beta)^2 B_2\epsilon^{(a-j\beta)t} + g_3^2 B_3 \epsilon^{g_3 t}\right] = 0 \tag{27}$$

Since the coefficients of $\epsilon^{g_1 t}, \epsilon^{g_2 t}$ and $\epsilon^{g_3 t}$ must separately add to zero if Eq. (27) is to be satisfied at all instants of times, we have

$$R_1(a+j\beta)A_1+(a+j\beta)^2A_1L_1+\frac{1}{C_1}A_1+M_pB_1(a+j\beta)^2=0 \quad (28)$$

$$R_1(a-j\beta)A_2+(a-j\beta)^2A_2L_1+\frac{1}{C_1}A_2+M_pB_2(a-j\beta)^2=0 \quad (29)$$

$$R_1g_3A_3 + L_1g_3^2A_3 + \frac{1}{C_1}A_3 + M_pB_3g_3^2 = 0 \quad (30)$$

For the instant $t = 0$ (the instant from which time is measured),

Let I_0 represent the value of the current i_1
I_p represent the value of the current i_p
E_{c0} represent the value of the voltage of the condenser e_c

Setting $t = 0$ and $i_1 = I_0$ in Eq. (26), there results

$$A_1 + A_2 + A_3 = I_0 - (i_s)_{t=0} = I_d \quad (30a)$$

The voltage of the condenser is

$$e_c = -\frac{q}{C} = -E + R_1[i_s + A_1\epsilon^{(a+j\beta)t} + A_2\epsilon^{(a-j\beta)t} + A_3\epsilon^{g_3t}]$$
$$+ L_1\left[\frac{di_s}{dt} + (a+j\beta)A_1\epsilon^{(a+j\beta)t} + (a-j\beta)A_2\epsilon^{(a-j\beta)t} + g_3A_3\epsilon^{g_3t}\right]$$
$$+ M_p\left[\frac{di_s}{dt} + (a+j\beta)B_1\epsilon^{(a+j\beta)t} + (a-j\beta)B_2\epsilon^{(a-j\beta)t} + g_3B_3\epsilon^{g_3t}\right] \quad (31)$$

Now

$$-(E)_{t=0} + (R_1i_s)_{t=0} + \left(L_1\frac{di_s}{dt}\right)_{t=0} + \left(M_p\frac{di_{ps}}{dt}\right)_{t=0} = (E_p)_{t=0} \quad (32)$$

Let

$$E_{c0} - (E_p)_{t=0} = E_{cd} \quad (33)$$

When $t = 0$, Eq. (31) becomes

$$E_{cd} = R_1I_d + L_1[(a+j\beta)A_1 + (a-j\beta)A_2 + g_3A_3] + M_p[(a+j\beta)B_1 + (a-j\beta)B_2 + g_3B_3] \quad (34)$$

Adding Eqs. (28), (29), and (30), there results

$$M_p[B_1(a + j\beta) + B_2(a - j\beta) + B_3g_3] = -R_1I_d - L_1[A_1(a + j\beta) + A_2(a - j\beta) + A_3g_3] - \frac{1}{C_1}\left[\frac{A_1}{a + j\beta} + \frac{A_2}{a - j\beta} + \frac{A_3}{g_3}\right] \quad (35)$$

Substituting Eq. (35) in Eq. (34), there results

$$E_{cd} = -\frac{1}{C_1}\left[\frac{A_1}{a + j\beta} + \frac{A_2}{a - j\beta} + \frac{A_3}{g_3}\right] \quad (36)$$

Neglecting the coefficient of ϵ^{g_3t}, we have

$$E_{cd} = -\frac{1}{C_1}\left[\frac{A_1}{a + j\beta} + \frac{A_2}{a - j\beta}\right] \quad (36a)$$

$$A_1 + A_2 = I_d \quad (37)$$

from which we have

$$A_1 = \frac{C_1E_{cd}(a^2 + \beta^2)}{2j\beta} + \frac{I_d(a + j\beta)}{2j\beta} \quad (38)$$

$$A_2 = -\frac{C_1E_{cd}(a^2 + \beta^2)}{2j\beta} - \frac{I_d(a - j\beta)}{2j\beta} \quad (39)$$

$$A_1\epsilon^{(a+j\beta)t} + A_2\epsilon^{(a-j\beta)t} = \epsilon^{at}[(A_1 + A_2)\cos\beta t + j(A_1 - A_2)\sin\beta t] \quad (40)$$

$$A_1 + A_2 = I_d \quad (41)$$

$$j(A_1 - A_2) = \frac{C_1E_{cd}(a^2 + \beta^2)}{\beta} + \frac{I_da}{\beta} \quad (42)$$

Since in most cases $R_1L_pG_p$ is small compared to $2L_1$ and D is very near unity, we may write

$$a = -\frac{R_1 - \beta^2h}{2L_1} \quad (43)$$

Then the expression for the current in circuit 1 becomes

$$i_1 = \frac{E}{\sqrt{(R_1 - \omega^2h)^2 + X_n^2}}\cos(\omega t - \tau - \lambda) + \left[I_d\cos\beta t + \left(\frac{C_1E_{cd}(a^2 + \beta^2)}{\beta} + \frac{I_da}{\beta}\right)\sin\beta t\right]\epsilon^{at} \quad (44)$$

In Eq. (44) if a^2 is neglected compared to β^2 and the value of a is written in as given in Eq. (43), Eq. (44) becomes

$$i_1 = \frac{E}{\sqrt{(R_1 - \omega^2 h)^2 + X_n^2}} \cos (\omega t - \tau - \lambda) + \left[I_d \cos \beta t + \left(C_1 \beta E_{cd} + \frac{a}{\beta} I_d \right) \sin \beta t \right] \epsilon^{-\frac{R_1 - \beta^2 h}{2L_1} t} \quad (45)$$

in which ω represents the impressed angular velocity.

Multiplying Eq. (29) by $\epsilon^{(a+j\beta)t}$, Eq. (30) by $\epsilon^{(a-j\beta)t}$, and Eq. (40) by $\epsilon^{g_3 t}$ and adding and substituting the result in Eq. (31), there results

$$e_c = -E + R_1 i_s + L_1 \frac{di_s}{dt} + M_p \frac{di_{ps}}{dt} - \left(\frac{A_1}{C_1(a + j\beta)} = D_1 \right) \epsilon^{(a+j\beta)t} - \left(\frac{A_2}{C_1(a - j\beta)} = D_2 \right) \epsilon^{(a-j\beta)t} + \frac{A_3}{C_1 g_3} \epsilon^{g_3 t} \quad (46)$$

Neglecting the coefficient of $\epsilon^{g_3 t}$, we have

$$D_1 + D_2 = -E_{cd} \quad (47)$$

$$j(D_1 - D_2) = \frac{E_{cd} a}{\beta} + \frac{I_d}{C_1 \beta} \quad (48)$$

The condenser voltage is

$$e_c = \frac{E}{\omega C_1 \sqrt{(R_1 - \omega^2 h)^2 + X_n^2}} \cos \left(\omega t + \frac{\pi}{2} - \tau - \lambda \right) + \left[E_d \cos \beta t - \left(\frac{E_{cd} a}{\beta} + \frac{I_d}{C_1 \beta} \right) \sin \beta t \right] \epsilon^{at} \quad (49)$$

APPENDIX C

TABLE OF SYMBOLS

The numbers when given refer to the section in which the symbol is introduced or in which it is most used.

A represents an amplification factor (8).

a represents the area in square centimeters of an antenna network (29).

a represents the real part of a generalized damping coefficient in Appendix B

A_f represents the power abstractive factor of an antenna (29).

B represents the modulating and demodulating constant of a triode. It is defined by the equation $\sqrt{i_p} = A + Be_g$ (48).

B_w represents the transmitted frequency band width (39)

b_1 and b_2 are coefficients in the Taylor expansion of the variable plate current as a function of the variable grid voltage (48).

C represents capacitance. It is generally used with subscripts to denote particular capacitances

C_{gf} represents the grid-to-filament capacitance of a triode (53).

C_{pf} represents the plate-to-filament capacitance of a triode (53)

C_{gp} represents the plate-to-grid capacitance of a triode (53).

D represents $1 + R_pG_p$ (11). Appendix B.

D represents distortion in telephone reception (41).

E represents voltage.

E_a and e_a represent the voltage introduced into a circuit by a resistance neutralizer (12, 13, 14, 15).

E_g represents the grid voltage, generally the r m s. value of the grid alternating voltage

E_p represents the plate voltage, generally the r m s value of the plate alternating voltage

E_{gp} represents the continuous grid voltage at the operating point

E_{pp} represents the continuous plate voltage at the operating point

F_m represents the peak value of the electric intensity in volts per centimeter (29)

f represents frequency in cycles per second

f_c represents the frequency of the correspondent station (27)

f_c represents the cutoff frequency of a radio receiving system (33)

f_i represents the frequency of the interferent station (27)

f_r represents the resonant frequency of a circuit

g represents a generalized damping coefficient (54). Appendix B.

G_{cg} represents the controlled grid conductance (4).

G_g represents the grid conductance (4).

G_{cp} represents the controlled plate conductance (4)

G_p represents the plate conductance (4).
h represents a resistance neutralization factor except in Sec. (29) where it represents the height of an antenna network in centimeters.
I represents current.
I_1 represents the r m.s. value of the current in the main oscillating circuit.
I_g represents the grid current, generally the r m.s. value of the grid alternating current.
I_p represents the plate current, generally the r.m s. value of the plate alternating current.
I_{pp} represents the continuous plate current at the operating point.
i_p represents plate space current.
j represents the rotative operator $\sqrt{-1}$.
k represents the coefficient of magnetic coupling (8).
k in Sec. 51 represents the modulating constant of the system.
k represents $\frac{R_r + R_w}{R_r}$ (29). Chap. V in general.
L represents self-inductance.
M represents mutual inductance.
N represents the amount by which the neutralizer lowers the resistance of a circuit (12).
N number of sparks per second (32).
n_r is defined by the equation $\frac{2}{r\pi}$ (31).
P represents power in watts. Various subscripts are used to designate various powers.
p represents the voltage transformation ratio of an ideal transformer (49)
p_1 and p_2 represent circuit parameters defined by

$$\mathbf{E}_a = (p_1 + jp_2)\mathbf{I}_p.$$

p_d represents the decimal parts of the resonant frequency of a circuit by which the interferent station is detuned (27).
p_o represents the permittivity of free space. It equals 8.85×10^{-14}.
p represents the frequency spacing of static generators (35)
$2pT$ represents the Morse space interval (31).
$2pT$ represents the time interval between static impulses (34)
q in general represents quantity of charge.
$2qT$ represents the time duration of a Morse dot (31).
$2qT$ represents the time duration of a static impulse (34).
R in general represents resistance.
R_d represents detector resistance.
R_p represents the external resistance in the plate circuit.
R_r represents radiation resistance.
R_n represents the net effective resistance of a circuit.
R_w represents the wasteful resistance of an antenna circuit
S_c represents the selective coefficient of a system (27).
s represents the velocity of light in free space. It equals 3×10^{10} centimeters per second (29).

T represents a time interval equal to one-half the time of a Morse dot and space (31).
T represents the time duration of a spark in spark telegraphy (32).
T_1 amd T_2 represent temperatures (2).
T_c represents the time constant of a circuit (27).
U represents a circuit parameter defined by $E_p = UI_1$ (15).
U_1 and U_2 represent circuit parameters defined by the equation

$$\mathbf{E}_p = (U_1 + jU_2)\mathbf{I}_1 \ (15).$$

V_1 and V_2 represent circuit parameters defined by the equation

$$\mathbf{E}_g = (V_1 + jV_2)\mathbf{I}_1 \ (15).$$

v_g represents the total grid voltage (2, 48).
v_p represents the total plate voltage (2, 48).
X represents reactance.
X_a represents the reactance introduced into a circuit by the resistance neutralizer (14).
X_c represents capacity reactance.
X_L represents inductive reactance.
W_s represents the static energy level (40, 41).
W_t represents the telephonic energy level (41).
Z represents impedance.
α represents a damping coefficient (32) Appendix A.
β represents the natural angular velocity of a circuit (20). Appendix B
β represents the ratio of reactance at the end of the transmitted band to the resistance at resonance (39).
γ represents $\frac{R - N}{R}$, the resistance reduction factor of a neutralizer when associated with a circuit of resistance R (12).
Δ represents an increment in the quantity following it
δ represents a circuit parameter defined by $\frac{E_p}{E_g} = \delta =$ [illegible]
δ represents the logarithmic decrement (32)
ρ represents a static amplitude factor, (34) only
ϵ represents the exponential base
ρ represents a circuit parameter defined by $R_d = \rho(R_r - R_a)$ (39)
μ represents the voltage amplification constant of a triode (2)
ω represents angular velocity
Ω_r represents natural angular velocity (11*a*). Appendix B
∞ represents infinity

INDEX

Numbers refer to sections, not to pages

A

E

F

G

H

I

L

M

N

O

P

R

S